

**The Hong Kong University of Science and Technology**

**UG Course Syllabus Template**

Course title: Number Theory and Applications

Course code: MATH 4141

Credits: 3

Prerequisite: MATH 2131

Co-requisite: MATH 3121

**Name:** Prof. MIN Yu

**Email:** maymin@ust.hk

**Office Hours:** by appointment

**Course Description**

This is a 4000-level course on Number Theory. It is designed simultaneously for pure math majors who are new to elementary number theory, as well as for advanced students who want in-depth knowledge about modern analytic approaches. The course is split into two parts.

1: The first part covers the essentials of elementary number theory, including modular arithmetic, quadratic forms, quadratic reciprocity, and quadratic number fields. This covers the development of modern number theory from Fermat to Gauss.

2: The second part is about analytic number theory, including arithmetic functions, the Primer Number Theorem, Dirichlet's Theorem on arithmetic progression and Dirichlet's analytic class number formula for quadratic fields.

Students are expected to have general knowledge of linear algebra, mathematical analysis, as well as minimal abstract algebra (basics of group, ring, field) and complex analysis (e.g. Cauchy's integral formula).

**Intended Learning Outcomes (ILOs)**

By the end of this course, students should be able to:

1. Be familiar with the history of modern number theory.
2. Be able to work with advanced methods in modern number theory.
3. Develop an understanding of the core ideas and concepts of algebraic number theory (in the quadratic case) and analytic number theory for further studies in advanced number theory courses.

## Assessment and Grading

This course will be assessed using criterion-referencing and grades will not be assigned using a curve. Detailed rubrics for each assignment are provided below, outlining the criteria used for evaluation.

### Assessments:

The assessment consists of 5 Homework, Midterm and Final Examination.

Homework: There will be 5 homework sets during the semester. Students should submit each homework in form of a neatly written and scanned PDF, or a LaTeX-typed PDF on the Canvas system before the deadline. All the questions will be graded.

Examinations: There will be a midterm exam on 18 October, 2025, and a final exam on 10 December, 2025.

The total point is calculated by the following formula with weights:

$$\begin{aligned} & \text{Total points} \\ &= HW(20\%) \\ &+ \begin{cases} \max_{W \in [10,40]} \{25, \text{Midterm} \times W\% + \text{Final} \times (80 - W)\%, & \text{if Midterm or Final} \geq 25 \\ \max(\text{Midterm}, \text{Final}) & \text{if Midterm and Final} < 25 \end{cases} \end{aligned}$$

Assessment Task	Contribution to Overall Course grade (%)	Due date
Homework	20%	TBC
Midterm Exam	10-40%	18/10/2025
Final Exam	40-70%	10/12/2025

Assessment marks for individual assessed tasks will be released within two weeks of the due date.

### Mapping of Course ILOs to Assessment Tasks

Assessed Task	Mapped ILOs	Explanation
Homework	ILO1, ILO2, ILO3	This task assesses students' understanding of the course materials (ILO 1,2,3).
Midterm Exam	ILO1, ILO2, ILO3	This task will assess students' understanding of the course materials (ILO 1,2,3), mainly focusing on the elementary number theory.

### Grading Rubrics

Homework, Midterm exam, Final Exam: All questions in will have answers to compare with and mark the scores.

**Final Grade Descriptors:**

Grades	Short Description	Elaboration on subject grading description
A	Excellent Performance	Demonstrates a comprehensive grasp of both elementary and analytic number theory, on both the conceptual understanding and computational aspects.
B	Good Performance	Shows good knowledge and understanding of both elementary and analytic number theory but did not master the subject completely.
C	Satisfactory Performance	Possesses adequate knowledge of core subject matter, mostly on elementary number theory.
D	Marginal Pass	The student has threshold knowledge of elementary number theory only, and able to complete some of the easiest computations.
F	Fail	The student does not have sufficient understanding of even some fragments of topics and is not even able to complete some of the easiest computations.

**Course AI Policy**

The use of Generative AI is permitted but students should make appropriate citation for its usage. Students should be critical of the response generated by AI and do not blindly copy the generated responses to their homework.

**Communication and Feedback**

Assessment marks for individual assessed tasks will be communicated via Canvas within two weeks of submission. Feedback on assignments will include comments and corrections. Students who have further questions about the feedback including marks should consult the instructor or TA within five working days after the feedback is received through email.

**Late submission Policy**

To ensure fairness for students who submit assignments on time, a penalty for late submission is listed as follows, according to the timestamp of Canvas:

- 1: Late submission between 0 to 24 hours, 50% penalty will be applied.
- 2: Late submission for more than 24 hours will not be accepted.

**Required Texts and Materials**

- 1: Lecture notes by the Instructor and Prof. IP Ivan.
- 2: Primes of the form  $x^2 + ny^2$ : Fermat, Class Field Theory, and Complex Multiplication, by David A. Cox
- 3: A history of abstract algebra, by Jeremy Gray
- 4: Number Theory: An approach through history from Hammurapi to Legendre, by André Weil
- 5: A course of elementary number theory, by Robert C. Vaughan.

6: Introduction to Analytic Number Theory, by Tom Apostol

7: A Course in Arithmetic, by Jean-Pierre Serre.

### **Academic Integrity**

Students are expected to adhere to the university's academic integrity policy. Students are expected to uphold HKUST's Academic Honor Code and to maintain the highest standards of academic integrity. The University has zero tolerance of academic misconduct. Please refer to [Academic Integrity | HKUST – Academic Registry](#) for the University's definition of plagiarism and ways to avoid cheating and plagiarism.