

MATH 4151 – Introduction to Lie Groups

Fall 2025-2026 Course Outline

Lecture

Instructor: Prof. IP, Ivan Chi-Ho
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Office Hour: By appointment
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Tutorials

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Time and Venue

Lecture: Monday, Wednesday 12:00 – 13:20 Room 4504 (Lift 25-26)

Session T1A: Wednesday 15:00 – 15:50 Room 5583 (Lift 27-28)

Course Description

This is an introduction to Lie groups, and is mainly divided into two parts:

Matrix Groups: Examples of Matrix Groups, Exponential Maps, Lie Algebras, Spin Groups

Lie Groups: Review of Manifolds, Smooth Structures, Cartan Closed Subgroup Theorem, Homogeneous Spaces, Tori and Heisenberg Groups.

Knowledge about Manifolds is helpful but not required.

This course will serve as a prerequisite for MATH5143 (Lie Algebra), so the exposition to the general theory of Lie Algebra will be kept minimal.

Prerequisite: (MATH2043 OR MATH3033) AND MATH2131 AND (3121 OR 3131)
Credits: 3

Intended Learning Outcomes

Upon completion of this course, students are expected to be able to

1. Explain and illustrate the fundamental principles and structures of Lie groups.
2. Convert and relate Lie groups to their corresponding Lie algebras.
3. Utilize analytic methods and computational techniques to solve problems involving matrix groups.
4. Analyze and interpret key structural results and theorems related to abstract Lie groups.

Assessment and Grading

The assessment consists of 5 Homeworks, Midterm and Final Examination.

Homework. There will be 5 Homework Sets. Students should submit each homework in the form of a LaTeX-typed PDF, or a clearly scanned handwritten PDF on the Canvas system before the deadline.

Examinations. There will be a 3-hour midterm exam during Week 6-8 (exact date to be confirmed), and a 3-hour final exam arranged by ARO.

Grading Scheme. This course will be assessed using **criterion-referencing** and grades will **not** be assigned using a curve.

The grade is computed by taking the maximum of the weights such that the total is 100%:

	Weight	Assessment ILOs
Homework	20%	1,2,3,4
Midterm Examination	10–40%	1,2,3,4
Final Examination	40–70%	1,2,3,4

Grading Scheme

Letter grades will be assigned depending on overall performance.

Obtaining a total point of 90% or above, or top 10%, will be guaranteed an A+.

Obtaining a total point of 70% or above will be guaranteed an A-range.

Obtaining a total point of 50% or above will be guaranteed an B-range.

Obtaining a total point of 30% or above will be guaranteed a passing grade.

Grades	Short Description	Elaboration on subject grading description
A	Excellent Performance	Demonstrates a comprehensive grasp of the theory of Lie groups, on both the conceptual understanding and computational aspects.
B	Good Performance	Shows good knowledge and understanding of the theory of Lie groups, but did not master the subject completely.
C	Satisfactory Performance	Possesses adequate knowledge of matrix groups only.
D	Marginal Pass	The student is only able to recall some fragments of topics and is able to complete some of the easiest computations.
F	Fail	The student does not have sufficient understanding of even some fragments of topics, and is not even able to complete some of the easiest computations.

Academic Integrity

Students are expected to adhere to the university's academic integrity policy. Students are expected to uphold HKUST's Academic Honor Code and to maintain the highest standards of academic integrity. The University has zero tolerance of academic misconduct. Please refer to Academic Integrity — HKUST – Academic Registry for the University's definition of plagiarism and ways to avoid cheating and plagiarism.

Late submission Policy. To ensure fairness for students who submit assignments on time, a penalty for late submission is listed as follows, according to the timestamp of Canvas:

- Late submission between 0 to 24 hours, 50% penalty will be applied.
- Late submission for more than 24 hours will not be accepted.

Make-up Exam Policy. Make-up Midterm Exam will only be considered with at least a 4-hour notice and with proper support documents. Make-up Final will follow the make-up exam policy set by ARO.

Communication and Feedback. Assessment marks for individual assessed tasks will be communicated via Canvas within two weeks of submission. Feedback on assignments will include comments and corrections. Students who have further questions about the feedback including marks should consult the instructor or TA within five working days after the feedback is received through email or discord.

Course AI Policy. The use of Generative AI is permitted, but students should be critical of the responses generated by the AI and do not blindly copy the generated responses to your homework.

Student Learning Resources

Lecture Notes written by the instructor. **Lecture Slides** and **Lecture Videos** will also be available on canvas after class.

Additional References:

1. A. Baker, *Matrix Groups, An Introduction to Lie Group Theory*.
2. B. Hall, *Lie Groups, Lie Algebras, and Representations, An Elementary Introduction*.
3. J. Hilgert and K. H. Neeb, *Structure and Geometry of Lie Groups*.

Tentative Schedule

Part I	Matrix Groups
Lecture 1 – 5	Basics of Matrix Groups
Lecture 6 – 9	Exponential Map
Lecture 10 – 13	Lie Algebras
Lecture 14 – 15	Spin Groups
Part II	Lie Groups
Lecture 16 – 19	Lie Groups as Manifolds
Lecture 20 – 22	Homogeneous Spaces
Lecture 23 – 24	Tori and Heisenberg Groups.