MATH 4223 - Differential Geometry

Fall 2025-26

Instructor: Aaron CHOW Email: chowtka@ust.hk
TA: Haofang SUN Email: hsunav@ust.hk

Class Time/Venue: Mondays and Wednesdays 09:00 - 10:20am at Room 4504

Tutorial Time/Venue: Tuesdays 06:00 - 06:50pm at Room 2304

COURSE OVERVIEW

This course provides an introduction to the differential geometry of curves and surfaces in Euclidean space, with a focus on the notions of curvature. Topics include computations in local coordinate charts, the first and second fundamental forms, covariant derivatives, Christoffel symbols, geodesics, variation formulae, and the Gauss-Bonnet theorem. We will also discuss the distinction between extrinsic and intrinsic geometry, in particular *Gauss' theorema egregium*.

PREREQUISITES

Multivariable Calculus (MATH 2023/2011) and Linear Algebra (MATH 2121/2131).

TEXTBOOK AND REFERENCES:

Lecture notes will be uploaded prior to each class and will serve as the primary reference for this course. An additional key reference is Prof. Frederick Fong's lecture notes (available on Canvas), specifically *Part I: Differential Geometry in Euclidean Space*, which we will follow closely. In addition, the following textbooks are recommended:

- 1. Differential Geometry: curves-surfaces-manifolds, Third Edition by Wolfgang Kühnel
- 2. Differential Geometry of Curves and Surfaces Manfredo P. do Carmo
- 3. A Comprehensive Introduction to Differential Geometry, Vol. 1, Third Edition by Michael Spivak

INTENDED LEARNING OUTCOMES (ILOs):

Upon completion of this course, students are expected to:

- 1. acquire workable knowledge on the fundamental concepts of regular curves and surfaces in Euclidean spaces;
- 2. acquire necessary background for further studies in differential geometry, general relativity, and related fields;
- 3. appreciate the beauty of differential geometry, especially the Gauss-Bonnet's Theorem.

HOMEWORK

There will be 6 problem sets assigned (bi-weekly) throughout the term. Each homework must be submitted as a clearly written and scanned PDF or a LaTeX-typed PDF via the Canvas system before the stated deadline. No late submissions will be accepted.

Collaboration with classmates is encouraged. Students may form a group of up to three members to work on the assignment jointly. In this case, a single copy of the homework should be submitted on behalf of the group, and the same score will be awarded to all team members.

EXAMINATIONS

The midterm exam will be held in class (80 mins) during Week 6-8 (exact date to be confirmed), and a

3-hour final exam arranged by ARO.

MAKE-UP MIDTERM POLICY

- Under any circumstance, students who are unable to attend the midterm exam will **not** be offered a make-up test that takes place after the regular exam session.
- For students who have valid reasons for missing the midterm (such as time crash with another midterm), the instructor may approve an early midterm, or assign the midterm marks according to the final exam performance.
- On the other hand, for students who miss the midterm without a valid reason, the midterm score will be regarded as 0. This includes self-claimed sickness without any medical statement.

GRADING SCHEME

This course will be assessed using **criterion-referencing**, and grades will **not** be assigned using a curve.

| | Scheme A | Scheme B | Adress ILOS |
|--------------|----------|----------|-------------|
| Homework | 20% | 20% | 1, 2, 3 |
| Midterm Exam | 30% | 5% | 1, 2, 3 |
| Final Exam | 50% | 70% | 1, 2, 3 |
| Course Total | 100% | 95% | |

The final grade will be determined by the **maximum** of the two schemes.

GRADE DESCRIPTION

| Grades | Short Description | Elaboration on subject grading description | |
|--------|--------------------------|---|--|
| A | Excellent Performance | The student has mastered almost all concepts and techniques | |
| | | of differential geometry taught in the course, has excellent | |
| | | understanding of the deepest content of the subject, and | |
| | | acquired workable knowledge for further studies of abstract | |
| | | manifolds, Riemannian geometry, general relativity, and | |
| | | related fields. | |
| В | Good Performance | The student has mastered most computational techniques of | |
| | | differential geometry taught in the course, yet the understanding | |
| | | of some challenging concepts may not be deep enough for further | |
| | | studies on related advanced subjects. | |
| С | Satisfactory Performance | The student meets the minimum expectation of the instructor, | |
| | | has acquired some basic computational techniques of the subject, | |
| | | yet some concepts were not clearly understood. | |
| D | Marginal Pass | The student is only able to recall some fragments of topics | |
| | | and is able to complete some of the easiest computations. | |
| F | Fail | The student does not have sufficient understanding of even some | |
| | | fragments of topics, and is not even able to complete some of the | |
| | | easiest computations. | |

AI POLICY

Students are allowed to consult any person (including the instructor, TA, classmates, friends outside HKUST) in any homework for ideas and hints, but are required to write up the solutions by themselves. You are required to list the persons and references you have consulted in every homework.

The use of ChatGPT or other generative AI is allowed, and they are regarded as "persons" you have consulted, and therefore must be listed in your homework.

However, please be warned that at the current stage of development of AI, the response to problems in advanced courses – especially those in pure mathematics – is not quite reliable. Students should be

critical of the response generated by AI and do not blindly copy the generated responses to your homework.

ACADEMIC INTEGRITY

Students are expected to adhere to the university's academic integrity policy. Students are expected to uphold HKUST's Academic Honor Code and to maintain the highest standards of academic integrity. The University has zero tolerance of academic misconduct.