MATH 4033 - Calculus on Manifolds

Spring 2025-26

Instructor:	Aaron CHOW	**Email:**	chowtka@ust.hk
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Class Time/Venue:	Tuesdays and Thursdays 03:00 - 04:20pm at Room 2463		
Tutorial Time/Venue:	Mondays 03:00 - 03:50pm at Room 5402		

COURSE OVERVIEW

This course is an introduction to smooth manifolds. Topics include smooth maps between manifolds, tangent spaces and tangent maps, immersions/submersions and submanifolds, tensors and differential forms, exterior derivatives, integration on manifolds, the Generalized Stokes' Theorem, and an introduction to de Rham cohomology (deformation retracts and Mayer–Vietoris).

PREREQUISITES

Grade A- or above in MATH 2023 AND B- or above in MATH 2131.

TEXTBOOK AND REFERENCES:

Lecture notes will be uploaded prior to each class and will serve as the primary reference for this course. An additional key reference is Prof. Frederick Fong's lecture notes (available on Canvas), specifically *Part II: Differentiable Manifolds,* which we will follow closely. In addition, the following textbook is highly recommended:

- *Introduction to Smooth Manifolds*, by John M. Lee

INTENDED LEARNING OUTCOMES (ILOs):

Upon completion of this course, students are expected to:

1. acquire workable knowledge of the fundamental concepts of smooth manifolds, smooth maps, tangent spaces/tangent maps, and submanifolds;
2. develop fluency with tensors and differential forms on manifolds (wedge products, exterior derivatives) and apply them in computations and proofs;
3. appreciate the deep relationship between analysis, geometry, and topology, especially through the Generalized Stokes' Theorem and de Rham cohomology.

COURSE TOPICS

- **Abstract Manifolds.** Smooth manifolds; smooth functions and maps; tangent spaces and tangent maps; inverse function theorem; immersions/submersions; submanifolds.
- **Tensors and Differential Forms.** Cotangent spaces; tangent and cotangent bundles; tensor products; wedge products; exterior derivatives (and related operations as introduced in the notes).
- **Generalized Stokes' Theorem.** Manifolds with boundary; orientability; integration of differential forms; generalized Stokes' theorem.
- **De Rham Cohomology.** Closed vs. exact forms; de Rham cohomology; deformation retracts; Mayer–Vietoris theorem and basic computations/examples.

HOMEWORK

There will be 4-5 problem sets assigned throughout the term. Each homework must be submitted as a clearly written and scanned PDF or a LaTeX-typed PDF via the Canvas system before the stated deadline. **No late submissions will be accepted.**

Collaboration with classmates is encouraged. Students may form a group of up to three members to work on the assignment jointly. In this case, a single copy of the homework should be submitted on behalf of the group, and the same score will be awarded to all team members.

EXAMINATIONS

The midterm exam will be held during Week 6-8 (exact date to be confirmed), and a 3-hour final exam arranged by ARO.

MAKE-UP MIDTERM POLICY

- Under any circumstance, students who are unable to attend the midterm exam will **not** be offered a make-up test that takes place after the regular exam session.
- For students who have valid reasons for missing the midterm (such as time crash with another midterm), the instructor may approve an early midterm, or assign the midterm marks according to the final exam performance.
- On the other hand, for students who miss the midterm without a valid reason, the midterm score will be regarded as 0. This includes self-claimed sickness without any medical statement.

GRADE DESCRIPTION

Grade	**Short Description**	**Elaboration on subject grading description**
A	Excellent Performance	The student has mastered almost all concepts and techniques taught in the course, including smooth manifolds and smooth maps, tangent spaces and submanifolds, tensor calculus and differential forms, and can confidently apply Generalized Stokes' Theorem and basic de Rham cohomology tools. The student demonstrates strong problem-solving ability and is well-prepared for further studies in Riemannian geometry, geometric analysis, topology, general relativity, and related fields.
B	Good Performance	The student has mastered most core definitions and computational techniques (e.g. working with tangent/cotangent data, pull-backs, exterior derivatives, and basic applications of Stokes' theorem). Some more abstract or advanced topics (e.g. cohomology computations) may not yet be fully internalized for further independent study.
C	Satisfactory Performance	The student meets the minimum expectations of the instructor, and can carry out basic computations and apply standard definitions in familiar settings. However, some key concepts and proofs were not clearly understood, and performance may be inconsistent on more conceptual questions.
D	Marginal Pass	The student is only able to recall fragments of the topics and complete some of the easiest computations, but shows significant gaps in understanding and limited ability to solve problems that require combining ideas.
F	Fail	The student does not have sufficient understanding of even basic concepts of the course and is unable to complete some of the easiest computations.

GRADING SCHEME

This course will be assessed using **criterion-referencing**, and grades will **not** be assigned using a curve.

	Scheme A	**Scheme B**	Adress ILOS
Homework	20%	20%	1, 2, 3
Midterm Exam	30%	5%	1, 2, 3
Final Exam	50%	70%	1, 2, 3
Course Total	100%	95%	

The final grade will be determined by the **maximum** of the two schemes.

AI POLICY

Students are allowed to consult any person (including the instructor, TA, classmates, friends outside HKUST) in any homework for ideas and hints, but are required to write up the solutions by themselves. You are required to **list the persons and references** you have consulted in every homework.

The use of ChatGPT or other generative AI is allowed, and they are regarded as "persons" you have consulted, and therefore must be listed in your homework.

However, please be warned that at the current stage of development of AI, the response to problems in advanced courses – especially those in pure mathematics – is not quite reliable. Students should be critical of the response generated by AI and do not blindly copy the generated responses to your homework.

ACADEMIC INTEGRITY

Students are expected to adhere to the university's academic integrity policy. Students are expected to uphold HKUST's Academic Honor Code and to maintain the highest standards of academic integrity. The University has zero tolerance of academic misconduct.