香港科技大學
THE HONG KONG
UNIVERSITY OF SCIENCE
AND TECHNOLOGY

數學系
DEPARTMENT OF MATHEMATICS

11th HKUST Undergraduate Math Competition – Junior Level

April 25th, 2026

INSTRUCTIONS

- Do **NOT** turn over the page until instructed to do so.
- This is a **CLOSED BOOK, CLOSED NOTES** competition.
- Calculators are **NOT** allowed.
- Provide complete details of your solution. You may use results from **any UG courses** with proper citations.
- Each problem carries equal weight.

Time allowed: 3 hours.

Problem 1. What is the smallest positive integer $k \in \mathbb{N}$ such that the sum of the decimal digits of

$$k \times (10^{2026} - 1)$$

is bigger than 33333?

Problem 2. Let $\mathbf{A}$ be a real 2026×2026 square matrix such that

$$\text{rank}(\mathbf{A}) + \text{rank}(\mathbf{A} - \mathbf{I}) = 2026.$$

where $\mathbf{I}$ denotes the identity matrix.

(Recall that: $\text{rank}(\mathbf{A})$ equals the (real) dimension of the span of its column vectors.)

Find all possible eigenvalues of $\mathbf{A}^{2026}$.

Problem 3. Let $\mathcal{S}$ be a finite set with 2026 elements, and $A_1, A_2, ..., A_{2026} \subseteq \mathcal{S}$ be any subsets. What is the probability that

$$A_1 \subseteq A_2 \subseteq \cdots \subseteq A_{2026}?$$

(You may assume that all the A_k's are chosen uniformly from the power set $\mathcal{P}(\mathcal{S})$ of $\mathcal{S}$.)

Problem 4. Expand the following infinite product as a formal power series

$$\prod_{n=1}^{\infty}(1 + nx^{3^n}) := 1 + a_1x^{k_1} + a_2x^{k_2} + \cdots = 1 + \sum_{i=1}^{\infty} a_i x^{k_i}$$

where both $a_i > 0$ and $k_1 < k_2 < \cdots$ are positive integers. Find a_{2026}.

Problem 5. Let $f(x)$ be the rational function

$$f(x) := \frac{3x - x^3}{1 - 3x^2}.$$

Find $f^{2026}\left(\sqrt{5 + 2\sqrt{5}}\right)$, where $f^n(x) := \underbrace{(f \circ f \circ \cdots \circ f)}_{n}(x)$.

Problem 6. Let $f(x) = \sum_{n=1}^{2026} \frac{x^n}{n}$ and $g(x) = \sum_{n=1}^{2026} nx^n$. Evaluate

$$\lim_{x \to +\infty} \frac{g(x)}{x} \int_0^x e^{f(t) - f(x)} dt.$$

–End of Paper–