Midterm topics review

- Functions, inverse functions
- Limits, limit rules
- Secant slope of 2 points on a graph

Definition of derivative

$$\lim_{x \to a} \frac{f(x)-f(a)}{x-a}, \text{ or } \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

Tangent slope at a point $(a, f(a))$ on the graph.

Tangent line. $y = f(a) + f'(a)(x-a)$

- Some special but important limits

$$\lim_{h \to 0} \frac{\sin(h)}{h} = 1, \quad \lim_{h \to 0} \frac{1-\cos(h)}{h} = 0$$

- Basic derivatives

$$\frac{d}{dx}(x^r) = r x^{r-1}, \quad \frac{d}{dx}(\sin x) = \cos x, \quad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(e^x) = e^x, \quad \frac{d}{dx}(\ln(x)) = \frac{1}{x}, \quad \frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{x^2+1}$$

- Basic derivative rules

(i) sum/difference

(ii) product $(fg)' = f'g + fg'$

(iii) quotient $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g}$

(iv) chain rule. $f \circ g$ = composition $(f \circ g)(x) = f(g(x))$

$(f \circ g)'(x) = f'(g(x))\, g'(x)$. Alternatively $y = g(x)$, $z = f(y)$

then $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$.

(v) Derivative of inverse function. If $y = f(x)$

and inverse is $x = f^{-1}(y)$, then

$$\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$$

- Relation of graph of f to its derivative f'
- Continuity. $\lim_{x \to a} f(x) = f(a)$.
- Vertical asymptote of a graph and infinite limits

Horizontal asymptote of a graph and limit at $+\infty$ or $-\infty$

- Implicit differentiation. If variables x, y satisfy an equation, to find $\frac{dy}{dx}$, apply $\frac{d}{dx}$ to the equation. Use derivative rules. Solve for $\frac{dy}{dx}$ in terms of x and y.
- Uses of exponentials/logarithms.
- Physical meaning of derivative at input a as the instantaneous rate of change.

- Differentials. If at input a, the function f has value f(a), derivative $\left.\frac{dy}{dx}\right|_a = f'(a)$

then if we want to estimate value of f at $a+\Delta x$, the change Δy from $f(a)$ is approximately

$$\frac{\Delta y}{\Delta x} \doteq \left.\frac{dy}{dx}\right|_{x=a} = f'(a)$$

So $\Delta y \doteq f'(a)\Delta x$, $f(a+\Delta x) \doteq f(a) + \Delta y = f(a) + f'(a)\Delta x$