HKUST

MATH1013 Calculus I Name:____________________

Sample Final Exam Student ID:____________________

Exam Time: 3 hrs Tutorial Section:____________________

Directions:

- This is a closed book examination. No Calculator is allowed in this examination.
- DO NOT open the exam until instructed to do so.
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Tutorial Section in the space provided above, and also in the **Multiple Choice Item Answer Sheet** provided.
- **Write the color version of your exam paper in the box under the "Date:" item in the Multiple Choice Item Answer Sheet**. (Green/Orange/Yellow/White)
- DO NOT use any of your own scratch paper. Use only the scratch papers provided by the examination. Write also your name on every scratch paper you use, and do not take any scratch paper away from the examination venue.
- When instructed to open the exam, please check that you have **12** pages of questions in addition to the cover page.
- Answer all questions. Show an appropriate amount of work for each short or long problem. If you do not show enough work, you will get only partial credit.
- You may write on the backside of the pages, but if you use the backside, clearly indicate that you have done so.
- **Cheating is a serious violation of the HKUST Academic Code. Students caught cheating will receive a zero score for the examination, and will also be subjected to further penalties imposed by the University.**

Please read the following statement and sign your signature.

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature :

____

Question No.	Points	Out of
Q. 1-22		44
Q. 23		12
Q. 24		10
Q. 25		12
Q. 26		12
Q. 27		10
Total Points		100

Part I: Answer all of the following multiple choice questions.

Mark your answers on the Multiple Choice Item Answer Sheet provided.

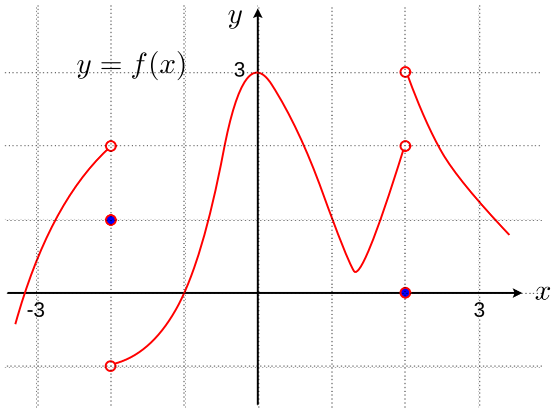
Do not forget to put your name, student ID number, and the color version of your exam paper on the Multiple Choice Item Answer Sheet.

Each of the following MC questions is worth 2 points. No partial credit.

1. For which constant k can the function $f(x) = \begin{cases} 4e^x + x - k & \text{if } x \leq 0 \\ \dfrac{\sin kx}{x} & \text{if } x > 0 \end{cases}$ be continuous everywhere?

   (a) 1 (b) 2 (c) 3 (d) 4 (e) None of the previous

2. Find $\displaystyle\lim_{x \to -2^-} \frac{|f(|x|) - 2|}{f(x) + 1}$ according to the given graph of f below.



   (a) 0 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$ (e) Does not exist

3. Find the horizontal asymptote of the function $y = \left(\cos \dfrac{1}{2x} - \dfrac{3}{x^2}\right)\left(1 + x \sin \dfrac{1}{x}\right)$.

   (a) 0 (b) 1 (c) 2 (d) 3 (e) Does not exist

4. A particle moves with position function given by $s(t) = 4t^3 - 6t^2 - 72t + 48$, for $t \geq 0$, where t is measured in seconds and s in meters. At what time is the particle at rest?

   (a) $t = 3$ sec (b) $t = 6$ sec (c) $t = 9$ sec (d) $t = 12$ sec (e) $t = 16$ sec

5. A mass attached to a spring has position function given by $s = 4\sin(3t)$, where s is the distance of the mass from its equilibrium position in centimeters, t is time in seconds. What is the acceleration of the mass at $t = \frac{\pi}{2}$?

   (a) 12 cm/s^2 (b) 24 cm/s^2 (c) 36 cm/s^2 (d) 48 cm/s^2 (e) 60 cm/s^2

6. Find $f'(1)$ where $f(x) = \dfrac{\cos \pi x}{x + \ln x}$

   (a) -2 (b) -1 (c) $\frac{1}{2}$ (d) 1 (e) 2

7. If $\dfrac{d}{dx}\left[f(\frac{1}{3}x^3)\right] = 2x^5$, what is $f'(x)$?

   (a) $3x$ (b) $6x$ (c) $3x^2$ (d) $2x^3$ (e) $\frac{2}{3}x^5$

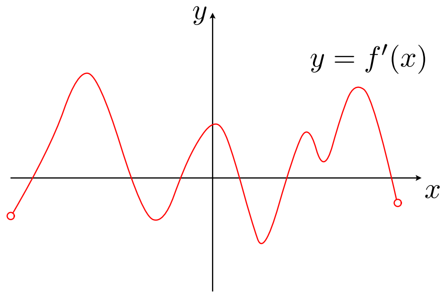
8. The increasing function $f(x) = x^3 + 4x - 2$ has an inverse function f^{-1}. Find the derivative $(f^{-1})'(-2)$.

   (a) $-\frac{3}{4}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{7}$ (d) $\frac{1}{4}$ (e) $\frac{1}{3}$

9. Find the slope of the tangent line to the curve defined by the equation $x^2(2-y) = y^3$ at the point $(1, 1)$.

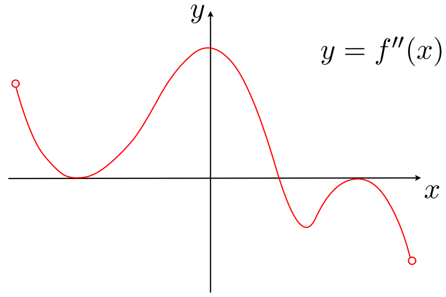
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) 3

10. Given the graph of the derivative function f' as shown below, exactly how many local maxima does f have?



(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

11. Given the graph of the second derivative function f'' as shown below, exactly how many inflection points does f have?

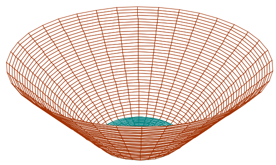


(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

12. Find the absolute minimum of the function $y = \cos(2x) + 2x\sin(2x)$ on the interval $0 \leq x \leq \pi$.

(a) 1 (b) -1 (c) $-\frac{\pi}{2}$ (d) $-\frac{2\pi}{3}$ (e) $-\frac{3\pi}{2}$

13. Water runs into a tank in the shape of a truncated regular cone with base radius 2 m, top radius 6 m and height 4 m at the rate of 10π m^3/min. How fast is the water level rising when the water is 3 m deep?



(a) 0.2 m/min (b) 0.4 m/min (c) 0.6 m/min (d) 0.8 m/min (e) 1 m/min

14. When applying the linear approximation (tangent line approximation) at $x = 0$ to approximate $f(0.02)$, where $f(x) = \sqrt{1+x} + \sin x$, the resulting approximate value is:

(a) 1.01 (b) 1.02 (c) 1.03 (d) 1.04 (e) 1.05

15. When applying Newton's method to find an approximate root of the equation $x^3 = 2$ with a starting value x_0, the resulting iteration formula is:

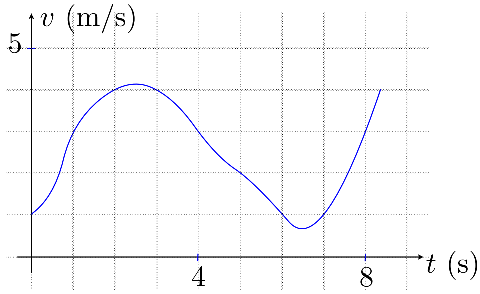
(a) $x_{n+1} = x_n - \frac{3x_n^2}{x_n^3-2}$ (b) $x_{n+1} = \frac{3x_n}{2} + \frac{3}{2x_n^2}$ (c) $x_{n+1} = \frac{2x_n}{3} - \frac{3}{2x_n^2}$

(d) $x_{n+1} = \frac{2x_n}{3} - \frac{2}{3x_n^2}$ (e) $x_{n+1} = \frac{2x_n}{3} + \frac{2}{3x_n^2}$

16. Find the limit: $\lim_{x\to 0}(e^x + 2x)^{\frac{1}{2x}}$.

(a) 1 (b) ∞ (c) $e^{\frac{1}{2}}$ (d) $e^{\frac{3}{2}}$ (e) $e^{\frac{1}{6}}$

17. The graph of the velocity function $v = v(t)$ of a travelling particle is given below. Use a midpoint Riemann sum over four subintervals of equal length to estimate the displacement of the particle over the time interval $0 \le t \le 8$, i.e., $s(8) - s(0)$ where $s = s(t)$ is the position function of the particle.



(a) 20 m (b) 22 m (c) 24 m (d) 26 m (e) 28 m

18. At time $t = 0$ minute, water starts flowing into an empty tank of volume 48 m^3 at a rate of $r(t) = 9\sqrt{t}$ m^3/min. How long will it take to fill the tank?

(a) 4 min (b) 6 min (c) 8 min (d) 10 min (e) 12 min

19. Evaluate the definite integral $\displaystyle\int_0^1 \sqrt{e^{3x+1}}\,dx$.

(a) $\frac{2}{3}(e^4 - e)$ (b) $\frac{3}{2}(e^4 - e)$ (c) $\frac{3}{2}(e^2 - e^{\frac{1}{2}})$

(d) $\frac{2}{3}(e^2 - e^{\frac{1}{2}})$ (e) $\frac{1}{3}(e^2 - e^{\frac{1}{2}})$

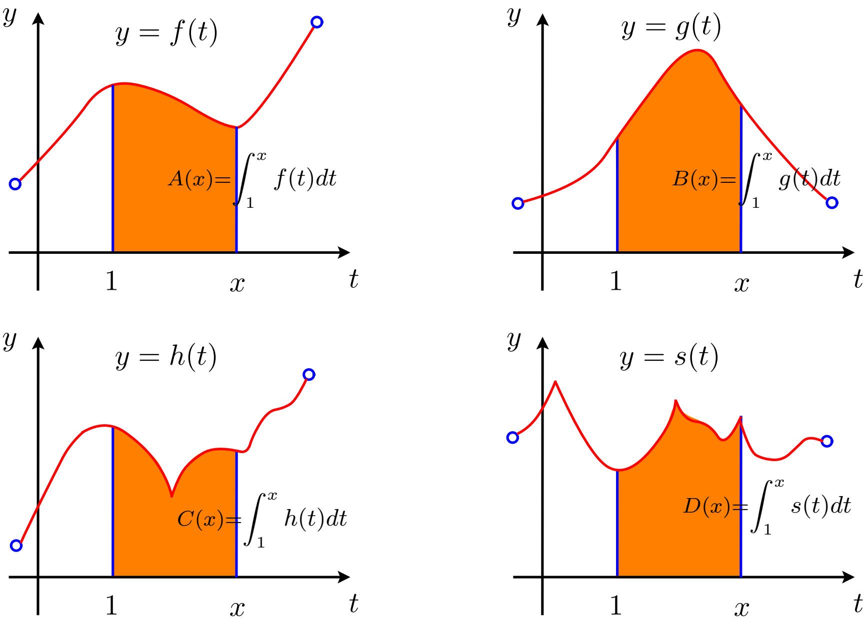
20. Evaluate the definite integral $\displaystyle\int_0^2 \frac{x^2}{\sqrt{1+x^3}}dx$.

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{6}{3}$ (d) $\frac{8}{3}$ (e) 3

21. Find the limit $\lim_{n\to\infty}\left(\frac{1}{2n+1}+\frac{1}{2n+2}+\cdots+\frac{1}{2n+n}\right)$ by computing a suitable definite integral.

(a) $\ln 2$ (b) $\ln 3$ (c) $\ln\frac{3}{2}$ (d) $\frac{1}{3}$ (e) $\frac{2}{3}$

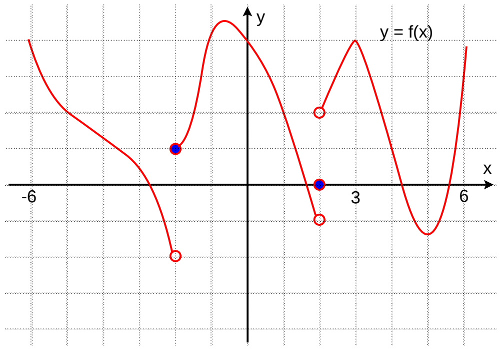
22. How many of the following area functions $A(x)$, $B(x)$, $C(x)$ and $D(x)$ for the given continuous functions as shown by the graphs below respectively can be non-differentiable at some x in the domain?



(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

Part II: Answer each of the following questions.

23. [*12 pts*] The graph of a function f is given as below, with points of discontinuity at only $x = -2$ and $x = 2$.



Determine the following limits.

(a) $\displaystyle\lim_{x\to 2^-} \frac{[f(x)]^2 + 3f(x) + 2}{[f(x)]^2 - 1}$ [4 pts]

(b) $\displaystyle\lim_{x\to -2} f^{-1}(x)$, where f^{-1} is the inverse function of f when restricted to the domain $-6 < x < -2$. [4 pts]

(c) $\displaystyle\lim_{x\to 0} \frac{\int_0^{x^2} (t-1)f(t)\,dt}{4x^2}$ [4 pts]

24. [10 *pts*] Consider the polynomial function $f(x) = x^5 - 10x^3 + 500$.

(a) Find all intervals of increase or decrease of f. (**No partial credit.**) [3 pts]

Interval(s) of increase: ____________________

Interval(s) of decrease: ____________________

(b) Find all concave up intervals of f. (**No partial credit.**) [3 pts]

Concave up interval(s) : ____________________

(c) How many real roots does f have? Justify your answer for full credit. [4 pts]

25. ([*12 pts*]) A function and its derivative are known as

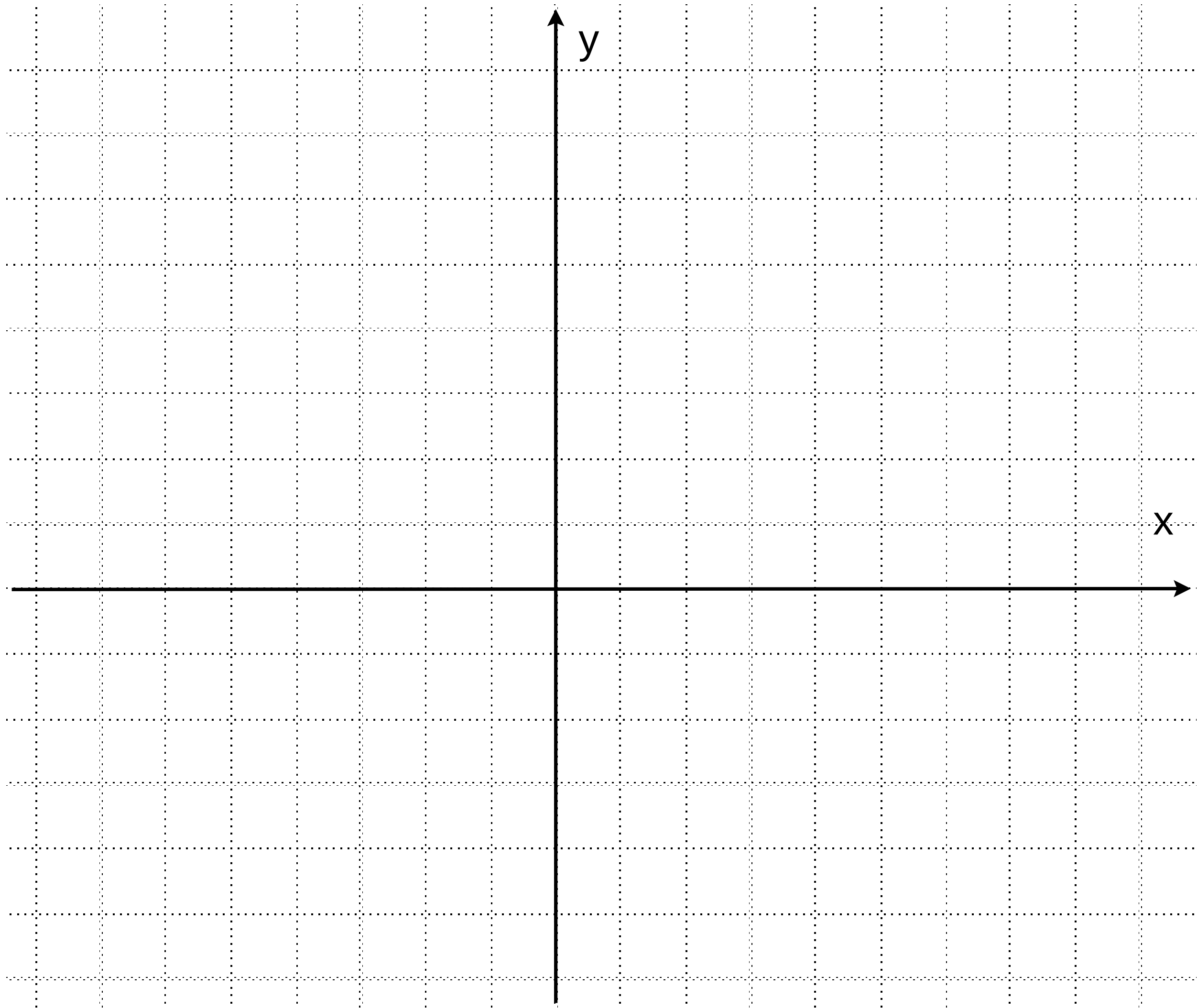
$$f(x) = \frac{3 - x^2 + 2x^3}{x^3}, \qquad f'(x) = \frac{x^2 - 9}{x^4}$$

Using appropriate scales on the axes in the given grid, sketch the following:

(a) All vertical and horizontal asymptotes of the function. [3 pts]

(b) All points on the graph of f with horizontal tangent line. [2 point]

(c) The graph of f, with all inflection point(s) clearly indicated together with their coordinates. [7 pts]



26. ([*12 pts*]) Three functions f, g, and h with continuous derivatives are given. Some function values of these functions are shown in the following table:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
0	2	0	3	-2	0	3
2	0	3	2	0	0	1
4	1	6	2	2	1	2

Answer the following questions:

(a) Evaluate the definite integral.

(i) $\displaystyle\int_0^2 \left[2f'(x) - 6xg'(x^2)\right]dx$ [4 pts]

$=$

(ii) $\displaystyle\int_{-1}^1 [f(x+3)]^4 f'(x+3)dx$ [4 pts]

$=$

(b) The graph of the function $y = 2f(x) - g(x) + h(x)$ must have a horizontal tangent line. [4 pts]

The statement above is : **True** **False** (Circle your answer.)
Brief reason:

27. [*10 pts*] A track field is to be built in the shape of a region bounded between two parallel lines and two semi-circles.

(a) If the perimeter of the field is exactly 5 km, find a design to maximize the rectangular area within it. In particular, what are the side lengths of the rectangular area of your design, and what is the maximum rectangular area thus obtained? [4 points]

(b) If you want to keep the maximum rectangular area obtained in part (a) in a track field of the required shape above, what is the smallest possible perimeter? [6 points]