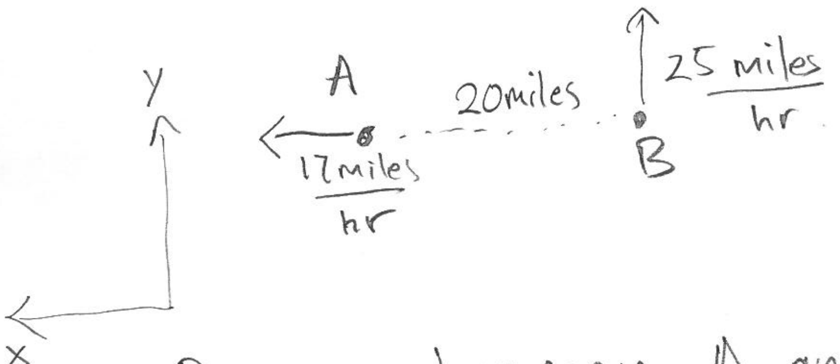
Related rates We have several quantities $Q_1, Q_2, \ldots$ which are functions of parameter s, and related by an equation. Equation allows us to find relation between $\frac{dQ_1}{ds}, \frac{dQ_2}{ds}$, etc. . .

WW5 #9 @ noon



How fast are ships separating at 7pm.

parameter is time t

Location of ship A is $(\underbrace{20+17t}_{x(t)}, 0)$

Location of ship B is $(0, \underbrace{25t}_{y(t)})$

Distance between A and B is

$t=0$ is noon.

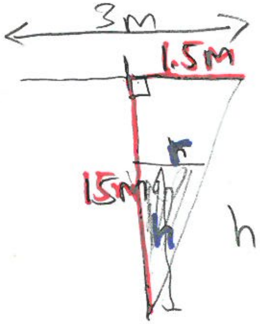
$$D = D(t) = \sqrt{(20+17t)^2 + (25t)^2} = \sqrt{x^2+y^2}$$

$$\left.\frac{dD}{dt}\right|_{t=7} = \frac{d}{dt}\left((x^2+y^2)^{1/2}\right) = \left.\frac{1}{2}(x^2+y^2)^{-1/2}\cdot\left(2x\cdot\frac{dx}{dt} + 2y\frac{dy}{dt}\right)\right|_{t=7}$$

$$= \cancel{\frac{1}{2}}\left((20+17\cdot7)^2 + (25\cdot7)^2\right)^{-1/2}\cdot\left(\cancel{2}(20+17\cdot7)17 + \cancel{2}\cdot25\cdot7\cdot25\right).$$

@ $t=7$, $x = 20+17\cdot7$, $y = 25\cdot7$

#10



diameter

radius.

h = height of water.

$\hookrightarrow$ water leaking @ 13,900 cm^3/min $\frac{dV_{out}}{dt}$

$V(t) = \frac{1}{3}\pi (r(t))^2 \cdot h(t)$.

$= \frac{1}{3}$ area base $\cdot$ height.

Now $\frac{r}{h} = \frac{150}{1500} \frac{cm}{cm}$

$r = \frac{1}{10} h$.

$V = \frac{1}{3}\pi \left(\frac{1}{10}h\right)^2 \cdot h$

$V = \frac{\pi}{300} h^3$.

Given $\frac{dh}{dt} = 26\ cm/min$ when $h = 3$ meters $= \underline{300\ cm}$.

$$\frac{dV}{dt} = \left(\frac{dV_{in}}{dt}\right) - \frac{dV_{out}}{dt} = \text{unknown} - 13{,}900\ cm^3/min.$$

Need to find $\frac{dV}{dt}$ when $\underline{h = 300\ cm}$. Then $\frac{dV_{in}}{dt} = \frac{dV}{dt} + 13{,}900\ cm^3/min$

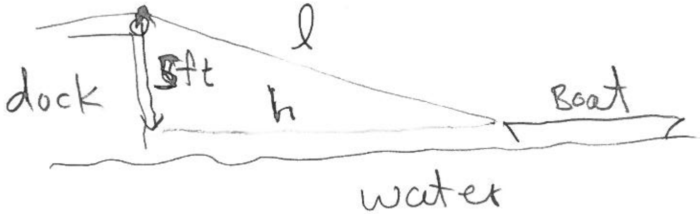
Know $\frac{dh}{dt} = 26\ cm/min$. Find relation between $\frac{dV}{dt}$ and $\frac{dh}{dt}$.

Since $V = \frac{\pi}{300} h^3$, we have $\frac{dV}{dt} = \frac{\pi}{300} \cdot 3h^2 \frac{dh}{dt}$.

@ $h = 300$ cm we get $\left.\frac{dV}{dt}\right|_{h=300cm} = \frac{\pi}{300} \cdot 3 \cdot (300)^2 \cdot \frac{26}{min}\ cm^3$.

$$\left(\frac{dV}{dt}\right)_{in} = \frac{\pi}{\cancel{300}_{100}} \cdot \cancel{3} \cdot (300)^2\ 26\ cm^3/min + 13{,}900\ cm^3/min$$

#11



$h(t)$ = horizontal distance of boat to dock

$l(t)$ = hypothenuse.

$$\boxed{l^2 = h^2 + 5^2}$$

$$\frac{dl}{dt} = -18 \text{ ft/min}$$

Find $\frac{dh}{dt}$ when $l = 90$.

$$\frac{d}{dt}\left(l^2\right) = \frac{d}{dt}\left(h^2 + 5^2\right)$$

$$2l \cdot \frac{dl}{dt} = 2h \cdot \frac{dh}{dt} + 0 \quad \text{so} \quad \frac{dh}{dt} = \frac{l}{h} \cdot \frac{dl}{dt}.$$

Know $l = 90$ ft

$\frac{dl}{dt} = -18$ ft/min.

Solve for h.

$$h = \sqrt{l^2 - 5^2}$$
$$= \sqrt{90^2 - 5^2}$$

$$\frac{dh}{dt} = \frac{90 \cancel{\text{ft}}}{\sqrt{90^2 - 5^2} \cancel{\text{ft}}} \left(-18 \text{ ft/min}\right).$$

Applications of logarithm and exponentials

WW6 #1 Bank pays quarterly interest (4 times a year).
Initial investment of 3500 dollars. Left to accumulate for 7 years (28 quarters). Amount at end of 7 years is 4854.2

What was the annual interest rate? Call it r. Then quarterly interest rate is $\frac{r}{4}$.

If initial amount is A_0, then after one quarter the amount is

$$A_1 = \underbrace{A_0}_{\text{initial}} + \underbrace{\left(\frac{r}{4}\right) A_0}_{\text{interest}} = A_0\left(1+\left(\frac{r}{4}\right)\right)$$

After 2 quarters the amount is $A_2 = A_1\left(1+\frac{r}{4}\right) = A_0\left(1+\frac{r}{4}\right)^2$

After 28 quarter the amount is $A_{28} = A_0\left(1+\frac{r}{4}\right)^{28}$.

We need to solve $4854.2 = 3500 \cdot \left(1+\frac{r}{4}\right)^{28}$ for r. $\Big\{ \frac{4854.2}{3500} = \left(1+\frac{r}{4}\right)^{28}$

Take logarithm of both sides. Can use any base. Use natural logarithm $\ln$

$$\ln\left(\frac{4854.2}{3500}\right) = \ln\left(1+\frac{r}{4}\right)^{28} = 28\ln\left(1+\frac{r}{4}\right) \quad \text{so} \quad \frac{1}{28}\ln\left(\frac{4854.2}{3500}\right) = \ln\left(1+\frac{r}{4}\right)$$

So $1+\frac{r}{4} = e^{\frac{1}{28}\ln\left(\frac{4854.2}{3500}\right)}$ so $\frac{r}{4} = e^{\frac{1}{28}\ln\left(\frac{4854.2}{3500}\right)} - 1$

$r = 4\cdot\left(e^{\frac{1}{28}\ln\left(\frac{4854.2}{3500}\right)} - 1\right)$.

Multiply by 100% to get %. $r = 4.6999885\ldots$

To nearest $\frac{1}{10}$ $r = 4.7\%$