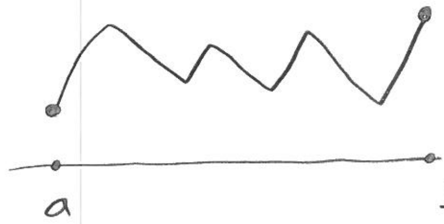
<u>Extreme Value Theorem (EVT)</u> If f is <u>continuous</u> on the <u>closed interval $[a,b]$</u>, then there is an input which gives absolute max, and there is an inpct which gives absolute min.



To find abs max/min we look at <u>set</u> of local max/min

<u>Local Extreme Value Theorem (LEVT)</u> If f continuous on $[a,b]$, then the local max/min occur among 3 sets

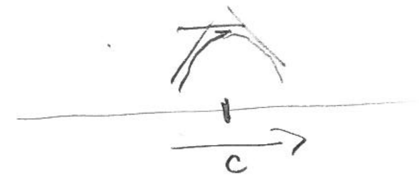
1. $f'(c) = 0$
2. $f'(c)$ DNE
3. endpoints.

(1) and (2): critical (numbers) points

To find abs max/min we use LEVT to find local max/min (usually <u>finite set</u>). From this finite set we can pick global/abs max/min

At a critical point where $f'(c) = 0$.

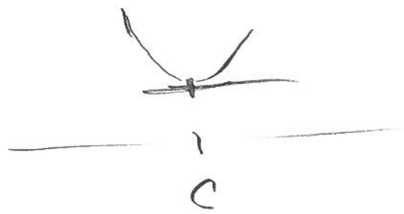
(1) If f' switches from $+$ to $-$ at c as x increases



$f''(c) < 0$

then input c yields local max

(2) If as x increases through c, the derivative switchs from $-$ to $+$



$f''(c) > 0$.

then local min.

<u>2nd derivative test</u>. If c is critical point $f'(c) = 0$

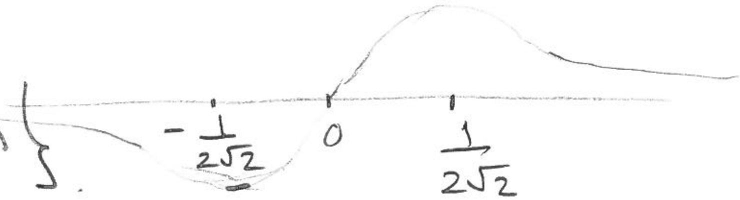
(2) If $f''(c) > 0$, then f' is <u>increasing</u> at c. So it must go from $-$ to $+$, which is local min

(1) If $f''(c) < 0$, then f' is <u>decreasing</u> at c. so f' must go from $+$ to $-$, which is local max

WWB #10 $\quad f(x) = 2x\,e^{-4x^2}$ $\quad$ domain $(-\infty, \infty)$

Note f is ODD, $f(0)$

(a) Find critical pt(s).



$$f'(x) = 2\cdot\left\{1\cdot e^{-4x^2} + x\cdot e^{-4x^2}(-4\cdot 2x)\right\}.$$

$$= 2e^{-4x^2}\left\{1 + x(-8x)\right\}$$

$$= 2e^{-4x^2}\left\{1 - 8x^2\right\}$$

$f' = 0$ when $1 - 8x^2 = 0$

$8x^2 = 1$

$x^2 = \frac{1}{8}$, $x = \pm\frac{1}{2\sqrt{2}}$

Use 2nd derivative test to determine nature of critical point

$$f''(x) = 2\left\{e^{-4x^2}(-4\cdot 2x)\{1-8x^2\} + e^{-4x^2}\{0 - 8\cdot 2x\}\right\}.$$

$$= 2e^{-4x^2}\left\{-8x + 64x^3 - 16x\right\}$$

$64x^3 - 24x$

$$= 2e^{-4x^2}(8x)(8x^2 - 3)$$

$(1 - 3)$

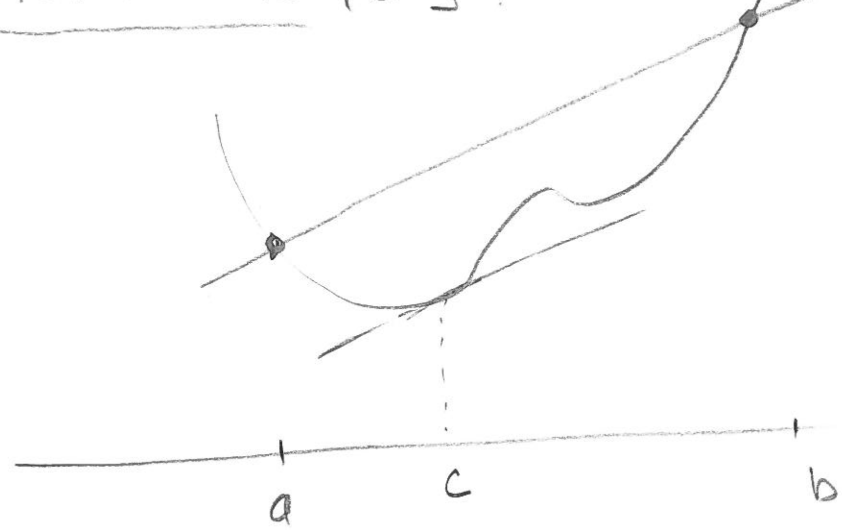
$$f''\left(-\tfrac{1}{2\sqrt{2}}\right) = 2e^{-4(\ )}(8)\left(-\tfrac{1}{2\sqrt{2}}\right)\left(8\cdot\tfrac{1}{8} - 3\right) > 0$$

2nd derivative test $\Rightarrow$ local min.

Since f odd $\Rightarrow$ f' even $\Rightarrow$ f'' odd.

$f''(-\frac{1}{2\sqrt{2}}) > 0 \Rightarrow f''(\frac{1}{2\sqrt{2}}) < 0 \Rightarrow$ local max.

Mean Value Theorem (MVT) Suppose f is differentiable on a closed interval $[a,b]$.



There is some point c in the interior so that tangent slope at $(c, f(c))$ equals the secant slope.

Uses of the mean value theorem:

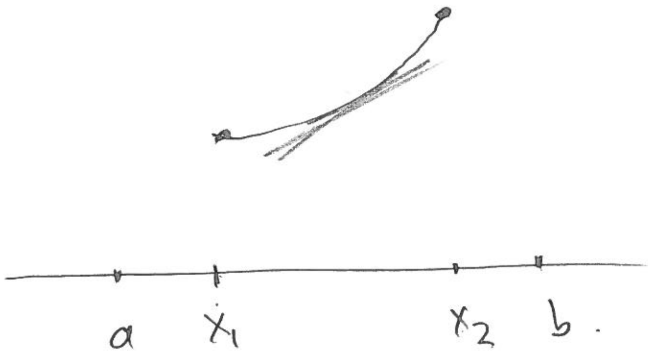
① Prove that $f' > 0 \Rightarrow f$ is increasing

$\qquad\qquad\qquad f' < 0 \Rightarrow f$ is decreasing

② $f' = 0$ on interval $[a,b] \Rightarrow f$ is constant.

Use of the MVT to show/prove that if $f' > 0$ on interval $[a,b]$, then f is increasing.

Imagine f on closed interval $[x_1, x_2]$.



The secant slope from $(x_1, f(x_1))$ to $(x_2, f(x_2))$

$$\frac{f(x_2)-f(x_1)}{x_2 - x_1} \overset{MVT}{=} \text{there is a point } c \text{ in interior } [x_1, x_2]$$

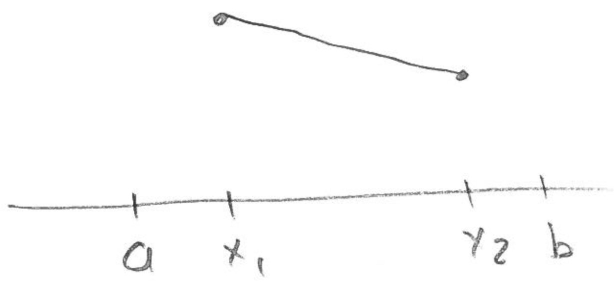
$$= f'(c). \quad (> 0 \text{ by hypothesis } f' > 0).$$

We can deduce that $f(x_2) - f(x_1) > 0$ so $f(x_1) < f(x_2)$.

Similarly if $f' < 0$.

Suppose f has 0 derivative on interval $[a,b]$

x_1, x_2 two points



$f(x_2$

Secant slope $(x_1, f(x_1))$ to $(x_2, f(x_2))$ is

$\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$ = MVT there is interior point c so

$= f'(c)$.

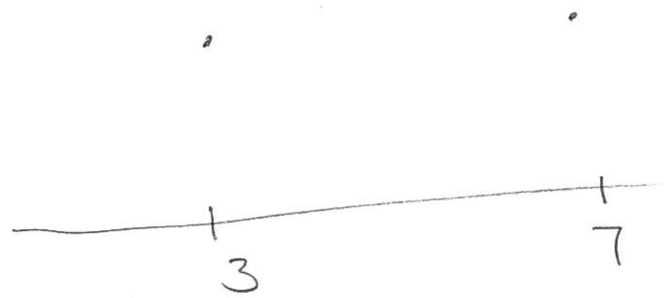
By hypothesis $f'=0$, we have $f'(c)=0$, so $f(x_1)=f(x_2)$

so f is constant.

WW6 #7. f continuous on $3 \le x \le 7$ (closed interval)

f differentiable with $-3 \le f'(x) \le 2$.

Use MVT to estimate $f(7) - f(3)$



Secant slope from $(3, f(3))$ to $(7, f(7))$ is

$$\frac{f(7)-f(3)}{7-3} = \text{secant slope} \overset{\text{MVT}}{=} f'(c).$$

there is interior point $c \in [3,7]$ so that

By hypothesis $-3 \le f'(x) \le 2$ we have $-3 \le f'(c) \le 2$

$$-3 \le \frac{f(7)-f(3)}{4} \le 2.$$

so $\quad -12 \le f(7) - f(3) \le 8$

The MVT follows (is proved) by using EVT and LEVT together.

The point c in the MVT occurs where the difference in $\underline{f(x)}$ and the secant line $\underline{s(x)}$ is maximal.

Look at difference

$$f(x) - s(x)$$

① By EVT has a max. say input c.

② By LEVT. tangent at c must equal secant slope.