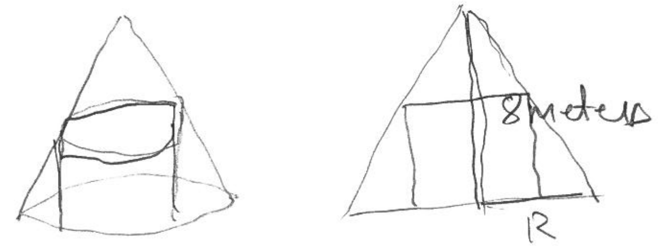
Optimatizion We have a differentiable (so continuous) function f, and we seek its max or min value.

EWW7 #1



Volume = 6 cubic meters

$$= \frac{1}{3} \text{ area base} \cdot \text{height}$$

$$= \frac{1}{3}\pi R^2 \cdot 8 \text{ m} = 6 \text{ m}^3$$

$$R^2 = \frac{9\cancel{18}}{4\cancel{8}\pi} \text{ m}^2$$

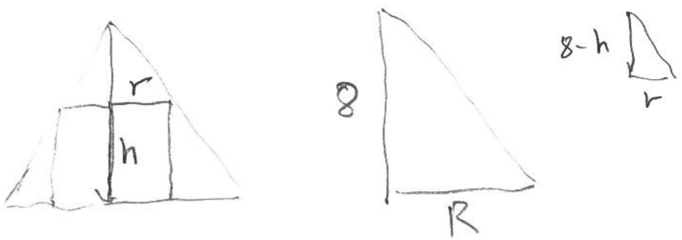
$$R = \sqrt{\frac{9}{4\pi}} \text{ meters.}$$

Inscribe a (~~inverted~~ cone) cylinder.

(1) Write formula for volume of cylinder.

(2) Find cylinder of max volume.

(1)



Relationship between h, r

$$\frac{8-h}{8} = \frac{r}{R} \quad \text{so} \quad 1 - \frac{h}{8} = \frac{r}{R}$$

solve for h to get $h = 8\left(1 - \frac{r}{R}\right)$

Volume cylinder = area base · height $= \pi r^2 \cdot h = \pi r^2\, 8\left(1 - \frac{r}{R}\right)$

$V(r) = 8\pi\, r^2\left(1 - \frac{r}{R}\right)$. with domain $0 \le r \le R$.

(2) So we want to max/min function

$V(r) = 8\pi r^2\left(1-\frac{r}{R}\right)$ on interval $0 \le r \le R$

At endpoints: $V(0)=0$, $V(R)=0$

$r^2\left(1-\frac{r}{R}\right) = r^2 - \frac{r^3}{R}$

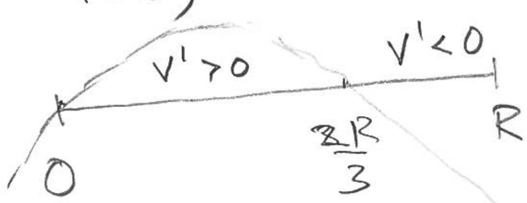
Max must occur at inside critical pt.

$V'(r) = (8\pi)\left(2r - \frac{3r^2}{R}\right) = (8\pi)\, r\left(2-\frac{3r}{R}\right)$

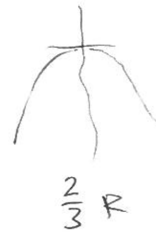
$= 0$ when $r=0$,

$2-\frac{3r}{R} = 0$

$\frac{2}{3} = \frac{r}{R}$ $\quad r = \frac{2}{3}R$



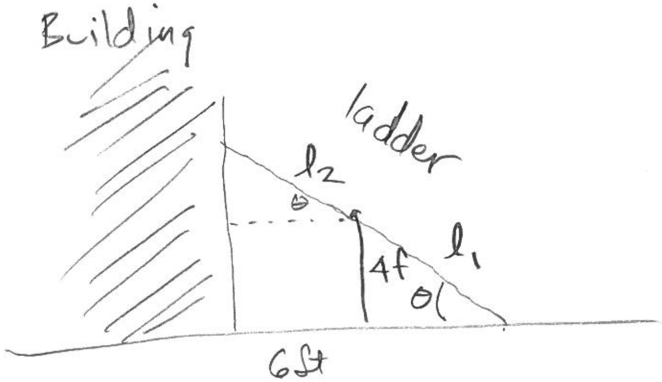
The critical point $\frac{2}{3}R$ yields local max.



Since absolute max must occur at critical pt or at endpoint, we see $r=\frac{2}{3}R$ yield absolute max

$h = 8\left(1-\frac{r}{R}\right) = 8\cdot\left(1-\frac{2}{3}\right) = \frac{8}{3}$. altitude for max cylinder.

WW 7 #4



ladder length $l = l_1 + l_2$.

What angle should θ be to so the ladder from ground to building clears fence but is as short as possible.

(1) Find function which gives length of ladder as a function of θ

l_1 : 

$\sin(\theta) = \dfrac{4}{l_1}$, $\quad l_1 = \dfrac{4}{\sin(\theta)}$.

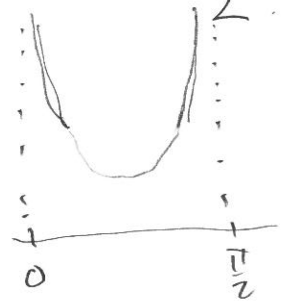
l_2 : 

$\cos(\theta) = \dfrac{6}{l_2}$, $\quad l_2 = \dfrac{6}{\cos\theta}$.

$$l(\theta) = \frac{4}{\sin\theta} + \frac{6}{\cos\theta}.$$

$$= 4(\sin\theta)^{-1} + 6(\cos\theta)^{-1}$$

$0 < \theta < \frac{\pi}{2}$.



But note as

$\theta \to 0^+$, $l(\theta) \to +\infty$

$\theta \to \frac{\pi}{2}^-$, $l(\theta) \to +\infty$

To find min length we find critical pt in $(0, \frac{\pi}{2})$

$$l'(\theta) = 4(-1)(\sin\theta)^{-2}(\cos\theta) + 6(-1)(\cos\theta)^{-2}(-\sin\theta)$$

$$= -4\frac{\cos\theta}{(\sin\theta)^2} + 6\frac{\sin\theta}{(\cos\theta)^2} = \frac{-4(\cos\theta)^3 + 6(\sin\theta)^3}{(\sin\theta)^2(\cos\theta)^2}$$

Critical pt $0 = -\frac{4\cos\theta}{(\sin\theta)^2} + 6\frac{\sin\theta}{(\cos\theta)^2}$ so $\frac{4\cos\theta}{(\sin\theta)^2} = \frac{6\sin\theta}{(\cos\theta)^2}$

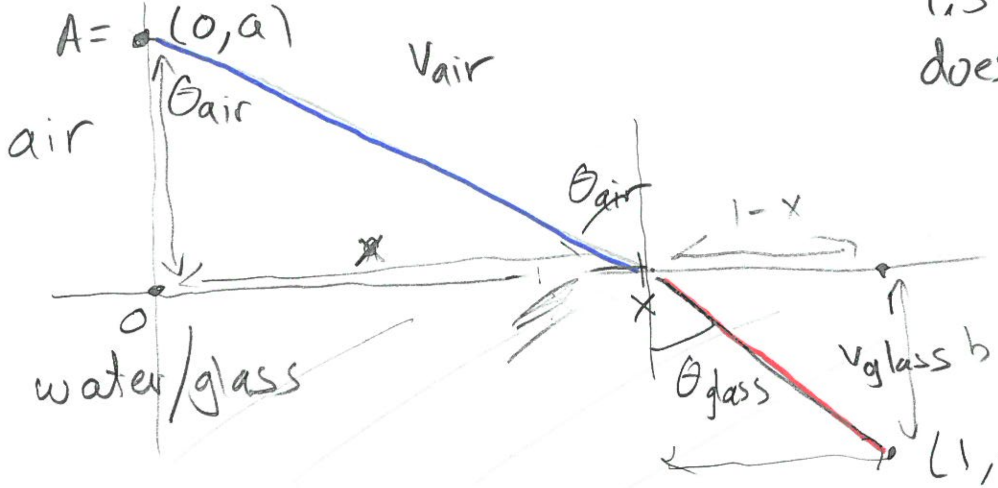
$$\frac{2}{3} = \left(\frac{\sin\theta}{\cos\theta}\right)^3$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \left(\frac{2}{3}\right)^{1/3}$$

so $\theta = \arctan\left(\left(\frac{2}{3}\right)^{1/3}\right)$. Expect local min (so abs min).

At critical point as θ increases $\cos\theta$ decreases, and $\sin(\theta)$ increases, so $-4(\cos\theta)^3 + 6(\sin\theta)^3$ switches from $-$ to $+$ so critical pt yield local min.

Example (Snell's law).



light travels in air 1.5 times as fast as it does in glass

$$v_{air} = 1.5\, v_{glass}.$$

(1) Write function in x of travel time from A=(0,a) to B=(1,-b).

$$T(x) = \frac{\sqrt{x^2+a^2}}{v_{air}} + \frac{\sqrt{(1-x)^2+b^2}}{v_{glass}} \qquad \text{domain } 0 \le x \le 1.$$

(2) Determine critical point

$$T'(x) = \frac{1}{v_{air}}\frac{1}{\cancel{2}}(x^2+a^2)^{-1/2}(\cancel{2}x+0) + \frac{1}{v_{glass}}\frac{1}{\cancel{2}}((1-x)^2+b^2)^{-1/2}\cdot(\cancel{2}(1-x)(-1))$$

$$0 = \frac{1}{v_{air}} \frac{x}{(x^2+a^2)^{1/2}} + (-1) \frac{1}{v_{glass}} \frac{(1-x)}{((1-x)^2+b^2)^{1/2}}$$

See $$\frac{1}{v_{glass}} \frac{(1-x)}{((1-x)^2+b^2)^{1/2}} = \frac{1}{v_{air}} \frac{x}{(x^2+a^2)^{1/2}}.$$

so $$1.5 = \frac{v_{air}}{v_{glass}} = \frac{\frac{x}{(x^2+a^2)^{1/2}}}{\frac{(1-x)}{((1-x)^2+b^2)^{1/2}}} = \frac{\sin \theta_{air}}{\sin \theta_{glass}} \quad (= 1.5)$$

<u>Snell's Law</u> $$\frac{v_{air}}{v_{glass}} = \frac{\sin \theta_{air}}{\sin \theta_{glass}}.$$

This law is the basis for eyeglasses.