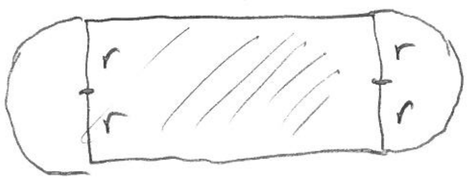
Week 13 Friday L03 2pm 

# #27

$\ell$




$2\pi r = 5$

(a) Perimeter is 5km $\quad P = 2\ell + 2\pi r = 5\text{ km}$. Solve $\ell$

$2\ell = 5 - 2\pi r$

Determine area of largest rectangle (shaded).

$\ell = \frac{5}{2} - \pi r$

$A = (2r)\,\ell = 2r\cdot\left(\frac{5}{2} - \pi r\right)$

$0 \le r \le \frac{5}{2\pi}$ (km)

$A = 5r - 2\pi r^2$

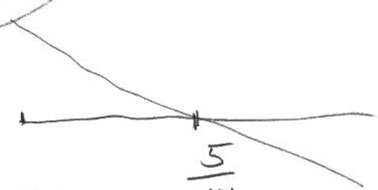
$A(0) = 0, \quad A\left(\frac{5}{2\pi}\right) = 0 \quad$ max NOT at endpoints

$A'(r) = 5 - 4\pi r$

$0 = A'(r) = 5 - 4\pi r$

$4\pi r = 5$

$r = \frac{5}{4\pi}$




$A'(0) > 0$

Absolute max at $r = \frac{5}{4\pi}$

$\ell = \frac{5}{2} - \pi\left(\frac{5}{4\pi}\right) = \frac{5}{2} - \frac{5}{4} = \frac{5}{4}$

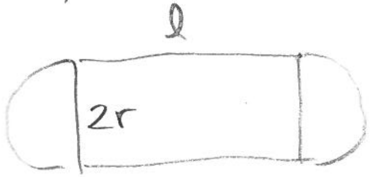
$A = 2\left(\frac{5}{4\pi}\right)\left(\frac{5}{4}\right) = \frac{25}{8\pi}$

$A'$ changes + to − at critical point

$\Rightarrow$ local max at $r = \frac{5}{4\pi}$

(b) Now keep $A = \frac{25}{8\pi}$. $A = \frac{25}{8\pi} = 2r \cdot l \rightarrow l = \frac{25}{16\pi}\frac{1}{r}$

Determine $r, l$ so Perimeter $P$ is minimum..




$$P = 2l + 2\pi r$$

$$P = 2 \cdot \left(\frac{25}{16\pi}\right)\frac{1}{r} + 2\pi r. \quad \text{domain } 0 < r < \infty.$$

$$\frac{dP}{dr} = 2\left(\frac{25}{16\pi}\right) \cdot (-1) r^{-2} + 2\pi$$

$\frac{dP}{dr} = 0$ when

$$2\left(\frac{25}{16\pi}\right)(-1) r^{-2} + 2\pi = 0$$

$$2 \cdot \left(\frac{25}{16\pi}\right)\frac{1}{r^2} = 2\pi$$

$$\frac{\cancel{2} \cdot 25}{16\pi \cdot \cancel{2}\pi} = r^2$$

$$\frac{25}{16\pi^2} = r^2$$

so $r = \frac{5}{4\pi}$.

This min

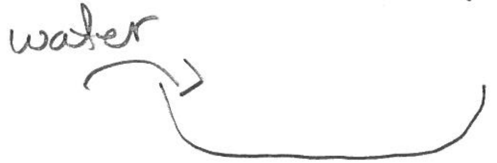
$$l = \frac{25}{16\pi}\frac{1}{r} = \frac{25}{\cancel{16}_{4}\cancel{\pi}} \cdot \frac{\cancel{4\pi}}{5}$$

$$l = \frac{5}{4}$$

same measurements (sizes) of $r, l$ as in part (a).

$\underline{P = 5 \text{ km}}$.

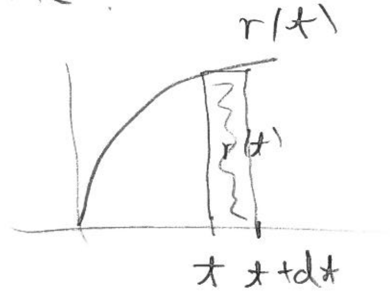
# #18 Definite integrals.




Tank  Volume of tank is $48\,(\text{meter})^3$.

<u>Rate</u> of water flowing into tank is $r(t) = 9\sqrt{t}\ \left(\frac{m^3}{min}\right)$

Starting $t=0$. How long to fill tank.




$\underline{r(t)\cdot dt}$ amount of water that follows in between $t$ and $t+dt$.

$$\int_0^{T_0} 9\sqrt{t}\,dt = 48 \qquad T_0 = \text{time to } \underline{\text{fill}} \text{ tank}$$

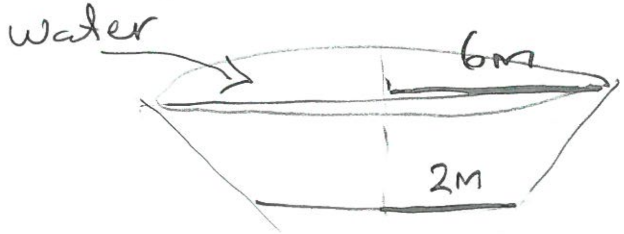
FTC II

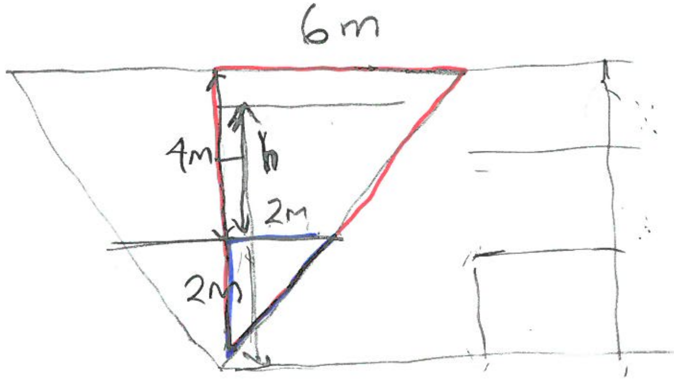
$$\left.\frac{9t^{3/2}}{(3/2)}\right|_0^{T_0} = 48 \quad \text{so} \quad \frac{3\cancel{9}\,(T_0)^{3/2}}{\cancel{3}/2} = 48$$

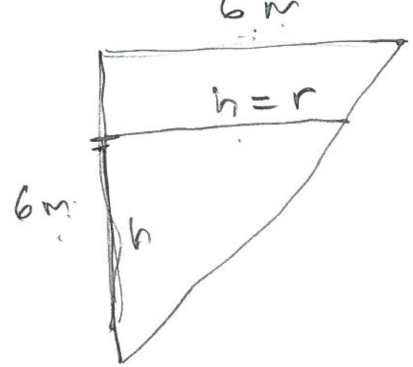
$$T_0^{3/2} = 8$$

$$T_0 = (8)^{2/3} = 2^2 = \underline{4 \text{ min}}$$

# #13 Related rates.




$$\frac{dV}{dt} = 10\pi \frac{(\text{meter})^3}{\text{min}}.$$

When water is 3 feet deep, find $\frac{dh}{dt}$.

"Add back cone". Find $\frac{dh}{dt}$ when $h = 3 + 2$

3ft above bottom. rest of cone

For the entire cone

$V = \frac{1}{3}$ area base · height

$$V = \frac{1}{3}\pi r^2 \cdot h = \frac{1}{3}\pi h^2 \cdot h = \frac{\pi}{3} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{\cancel{3}} \cancel{3} h^2 \cdot \frac{dh}{dt}$$

$$10\pi = \pi \cdot (3+2)^2 \cdot \frac{dh}{dt}$$

$$10\pi = 25\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{10\pi}{25\pi} = \frac{2}{5} = 0.4 \ \frac{\text{meters}}{\text{min}}.$$

# #21 Riemann Sums

Find $\lim_{n\to\infty} \left( \frac{1}{2n+1} + \frac{1}{2n+2} + \cdots + \frac{1}{2n+n} \right) = \ln\left(\frac{3}{2}\right)$.

$\frac{1}{2n+1} = \frac{1}{n} \cdot \frac{1}{(2+\frac{1}{n})}$, $\frac{1}{2n+2} = \frac{1}{n} \cdot \frac{1}{(2+\frac{2}{n})}$, ..., $\frac{1}{2n+n} = \frac{1}{n} \cdot \frac{1}{(2+\frac{n}{n})}$.

Suggests $n$ equal length subintervals of length $\frac{1}{n}$.

$\frac{1}{n} = \frac{b-a}{n}$ so $b-a=1$

Take $\underline{a=0}$. Then $\underline{b=1}$ What is function


$f\left(\frac{1}{n}\right) = \frac{1}{2+\frac{1}{n}}$, $f\left(\frac{2}{n}\right) = \frac{1}{2+\frac{2}{n}}$, ...

$f\left(\frac{n}{n}\right) = \frac{1}{2+\frac{n}{n}}$.

Guess $f(x) = \frac{1}{2+x}$. Sum is Right RS of $\int_0^1 \frac{dx}{2+x}$.

By FTC II $\int_0^1 \frac{dx}{2+x} = \ln(2+x)\Big|_0^1 = \ln(3) - \ln(2) = \ln\left(\frac{3}{2}\right)$

#22 FTC I Assumption If $f$ is continuous on $a \le x \le b$, then area function

$$A(x) = \int_a^x f(t)\,dt$$

is differentiable and $A'(x) = f(x)$.

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From 4 graphs the functions are ALL continuous, so ALL four area functions are differentiable

Zero are not differentiable

#10. Local <u>max</u> occurs at critical point ($f' = 0$)

where $f'$ changes sign <u>+ to −</u> at critical point

Graph of derivative $f'$ shows 3 critical points

where $f'$ sign change is + to −.

#11 Inflection point is where $f''$ changes sign.

From graph of $f''$, this occurs at 1 place.

# #14 Tangent line approximation

$$f(x) = \sqrt{1+x} + \sin x.$$

$$f'(x) = \frac{1}{2}(1+x)^{-1/2} + \cos x$$

$$f'(0) = \frac{1}{2}(1+0)^{-1/2} + \cos 0 = \frac{3}{2}$$

Note $f(0) = \sqrt{1+0} + \sin 0 = 1.$

Use tangent line to find approximation for $f(0.02)$

For tangent line need: $f(0) = 1,$
$f'(0) = \frac{3}{2}$

So $T_0(x) = f(0) + f'(0)(x-0) = 1 + \frac{3}{2}x$

$$T_0(0.02) = 1 + \frac{3}{\cancel{2}} \cdot 0.0\cancel{2} = 1 + 3 \cdot 0.01 = 1.03.$$