Instructions: Complete the following exercises.

Your work on the assigned problems will be graded on clarity of exposition as well as correctness. Feel free to discuss the problems with other students, but be sure to acknowledge your collaborators in your solutions, and to write up your final solutions by yourself.

Due in class on **Monday, April 24**.

Let (W, S) be a Coxeter system. Recall that definition of the *Kazhdan-Lusztig polynomials* $P_{yw} \in \mathbb{Z}[x^2]$ for $y, w \in W$ from class. Also recall the definitions of *left, right*, and *two-sided cells* from Lecture 20.

1. Prove that $P_{yw} = 1$ if $y \leq w$ and $\ell(w) - \ell(y) \leq 2$.
2. Suppose W is finite and w_0 is its longest element. Prove that $P_{y,w_0} = 1$ for all $y \in W$.
3. Prove that $P_{y,w} = P_{y^{-1},w^{-1}}$ for all $y, w \in W$.
4. Prove that 1 lies in a two-sided cell by itself. Prove that the same is true of w_0 if W is finite.
5. Compute the left, right, and two-sided cells in $W = S_4$, with $S = \{s_1, s_2, s_3\}$ where $s_i = (i, i+1)$.

   Look up the definition of the RSK correspondence $w \overset{\text{RSK}}{\longmapsto} (P, Q)$ (for example, here: `https://en.wikipedia.org/wiki/Robinson-Schensted-Knuth_correspondence`), and compute (or find a website online to compute) the pair of tableaux (P, Q) corresponding to each $w \in S_4$.

   How do these tableaux correspond to the division of S_4 into cells?

   Hint: First try this for S_3. You will need to know (at least the leading coefficients) of the polynomials P_{yw}. You are welcome to use the tables of these polynomials available online at

   `http://www.math.ias.edu/~goresky/tables.html`

   though it may take some effort to figure out how to interpret this data. If you want to calculate the KL polynomials yourself, feel free to do this with a computer program; just say what you did and include a copy of your code (or explain whose code you used) with your solutions.

   The Bruhat order on S_4 is shown below: