FINAL REVIEW – OPTIONAL ASSIGNMENT #1 – MATH 2121, FALL 2019.

Name:	
Student ID:	
Email:	

Lecture: L1 (Wed/Fri) L2 (Mon/Wed)

Tutorial: T1A T1B T1C T2A T2B T2C

These exercises are intended to help you review for our final examination.

This is the first of 4 optional review assignments.

If you bring your solutions to the assignments to the final, then we will grade your answers and you can earn up to 5 extra credit points on the exam.

You may use any available resources, including other people, but you should write up your answers individually.

Feel free to write your solutions on your own paper, but include all of the information requested on this title page and make sure to staple your answers together.

Exercise 1. Find a general formula for all solutions to the linear system

$$\begin{aligned} x_1 + 5x_3 &= 4 \\ 2x_1 + x_2 + 6x_3 &= 4 \\ 3x_1 + 4x_2 - x_3 &= -4 \end{aligned}$$

Solution:

Exercise 2. Express the vector $b = \begin{bmatrix} 2 \\ 13 \\ 6 \end{bmatrix}$ as a linear combination of the vectors

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \qquad v = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}, \qquad w = \begin{bmatrix} 5 \\ 6 \\ 0 \end{bmatrix}.$$

Solution:

Exercise 3. Show that the vector $b = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$ is not in the span of the vectors

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \qquad v = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}, \qquad w = \begin{bmatrix} 5 \\ 6 \\ -1 \end{bmatrix}.$$

Solution:

Exercise 4. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation with

$$T\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 5 \\ 6 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Find the standard matrix A for T, which satisfies $T(v) = Av$ for all $v \in \mathbb{R}^3$.

Solution:

Exercise 5. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a function

(a) Write down what it means for T to be *linear*.

(b) Write down what it means for T to be *one-to-one*.
Explain how to determine if T is one-to-one when T is linear.

(c) Write down what it means for T to be *onto*.
Explain how to determine if T is onto when T is linear.

Solution: