FINAL REVIEW – OPTIONAL ASSIGNMENT #2 – MATH 2121, FALL 2019.

Name:	
Student ID:	
Email:	

Lecture: L1 (Wed/Fri) L2 (Mon/Wed)

Tutorial: T1A T1B T1C T2A T2B T2C

These exercises are intended to help you review for our final examination.

This is the second of 4 optional review assignments.

If you bring your solutions to the assignments to the final, then we will grade your answers and you can earn up to 5 extra credit points on the exam.

You may use any available resources, including other people, but you should write up your answers individually.

Feel free to write your solutions on your own paper, but include all of the information requested on this title page and make sure to staple your answers together.

Exercise 1. Compute the matrix products

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

Solution:

Exercise 2. Find the inverse of $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 2 & 4 \\ 0 & 2 & 3 \end{bmatrix}$.

Solution:

Exercise 3. Write in your own words definitions to the following vocabulary:

(1) A *linear combination* of some vectors $v_1, v_2, \ldots, v_p \in \mathbb{R}^n$.
(2) The *span* of some vectors $v_1, v_2, \ldots, v_p \in \mathbb{R}^n$.
(3) A *linearly independent* set of vectors $v_1, v_2, \ldots, v_p \in \mathbb{R}^n$.
(4) A *linearly dependent* set of vectors $v_1, v_2, \ldots, v_p \in \mathbb{R}^n$.
(5) A *subspace* of $\mathbb{R}^n$.
(6) A *basis* of a subspace of $\mathbb{R}^n$.
(7) The *dimension* of a subspace of $\mathbb{R}^n$.
(8) The *column space* of a matrix A.
(9) The *null space* of a matrix A
(10) The *rank* of a matrix A.

Solution:

Exercise 4. Find bases for $\mathrm{Col}A$ and $\mathrm{Nul}A$ when $A = \begin{bmatrix} 6 & 3 & 6 & 9 \\ 4 & 2 & 4 & 6 \\ 6 & 3 & 5 & 9 \end{bmatrix}$.

Solution:

Exercise 5. Consider the matrix

$$A = \begin{bmatrix} 6 & 1 & 1 \\ 4 & -2 & 5 \\ 2 & 8 & 7 \end{bmatrix}.$$

(a) Compute $\det A$ using the formula

$$\det A = \sum_{X \in S_3} \mathsf{prod}(X, A)(-1)^{\mathsf{inv}(X)}.$$

(b) Compute $\det A$ using the row reduction algorithm discussed in Lecture 12.

(c) Compute $\det A$ using the formula

$$\det A = a_{11} \det A^{(1,1)} - a_{21} \det A^{(2,1)} + a_{31} A^{(3,1)}$$

discussed at the end of Lecture 12.

(d) Without doing any (significant) calculation, compute

$$\det A^{-1}, \qquad \det A^T, \qquad \det B, \qquad \text{and} \qquad \det C$$

for the matrices

$$B = \begin{bmatrix} 1 & 1 & 6 \\ 5 & -2 & 4 \\ 7 & 8 & 2 \end{bmatrix} \qquad \text{and} \qquad C = \begin{bmatrix} 12 & 1 & 2 \\ 8 & -2 & 3 \\ 4 & 8 & 15 \end{bmatrix}.$$

Solution: