FINAL REVIEW – OPTIONAL ASSIGNMENT #4 – MATH 2121, FALL 2019.

Name:	
Student ID:	
Email:	

Lecture: L1 (Wed/Fri) L2 (Mon/Wed)

Tutorial: T1A T1B T1C T2A T2B T2C

These exercises are intended to help you review for our final examination.

This is the third of 4 optional review assignments.

If you bring your solutions to the assignments to the final, then we will grade your answers and you can earn up to 5 extra credit points on the exam.

You may use any available resources, including other people, but you should write up your answers individually.

Feel free to write your solutions on your own paper, but include all of the information requested on this title page and make sure to staple your answers together.

Exercise 1.

(a) Draw a picture representing a subspace V, a vector b, and the orthogonal projection $\mathrm{proj}_V(b)$ of b onto V (say, in $\mathbb{R}^3$).

(b) Suppose A is an $m \times n$ matrix and $b \in \mathbb{R}^m$.
Assume the linear system $Ax = b$ is inconsistent.

Draw a picture representing $\mathrm{Col}A$ and b and $\mathrm{proj}_{\mathrm{Col}A}(b)$.

Use this picture to explain why the equation $Ax = \mathrm{proj}_{\mathrm{Col}A}(b)$ always has a solution and why a solution to this equation minimizes $\|Ax - b\|$.

(This shows that the exact solutions to $Ax = \mathrm{proj}_{\mathrm{Col}A}(b)$ are the least-squares solutions to $Ax = b$. We showed in class that the exact solutions to $Ax = \mathrm{proj}_{\mathrm{Col}A}(b)$ are the same as the exact solutions to $A^TAx = A^Tb$.)

Solution:

Exercise 2. There are three parts to this problem.

(a) Find an orthogonal basis for the column space of the matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 2 \\ 2 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix}.$$

(b) Find the orthogonal projection of the vector $v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ onto $\mathrm{Col}(A)$.

(c) Finally, find a basis for $\mathrm{Col}(A)^{\perp}$.

Solution:

Exercise 3. Suppose a function $f : \mathbb{R} \to \mathbb{R}$ has the following values:

x	$f(x)$
0	0
1	6
2	5
3	10
4	7

Find $a, b, c, d \in \mathbb{R}$ such that the cubic equation

$$y = ax^3 + bx^2 + cx + d$$

best approximates $f(x)$ in the sense of least-squares.

(This is related to #13 in the Final Review Problems on the course website.)

Solution:

Exercise 4. Consider the symmetric matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}.$$

Find an orthogonal matrix U and a diagonal matrix D such that

$$A = UDU^T.$$

(This is related to #14 in the Final Review Problems on the course website.)

Solution:

Exercise 5. Find a singular value decomposition for the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

(This is related to #15 in the Final Review Problems on the course website.)

Solution: