Instructions: *Complete the following exercises. Solutions will be graded on clarity as well as correctness. Feel free to discuss the problems with other students, but be sure to acknowledge your collaborators in your solutions, and to write up your final solutions by yourself.*

Due on **Tuesday, May 17**.

Let L denote a semisimple Lie algebra over an algebraically closed field $\mathbb{F}$ of characteristic zero.

1. Use character theory (namely, the results in §22.5 of the textbook) to show that if $L = \mathfrak{sl}_2(\mathbb{F})$ then $V(m) \otimes V(n) \cong V(m+n) \oplus V(m+n-2) \oplus \cdots \oplus V(m-n)$ for any nonnegative integers $n \leq m$. Here $V(m)$ denotes the irreducible $\mathfrak{sl}_2(\mathbb{F})$-module of dimension $m+1$.
2. Give a direct proof of Weyl's character formula when $L = \mathfrak{sl}_2(\mathbb{F})$.
3. Describe how to modify the bases of the classical Lie algebras given in §1.2 of the textbook to obtain Chevalley bases.
4. Suppose L is of type A_{n-1} and $\mathbb{K}$ is an extension of the finite field $\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$ (for $p > 0$ prime). Prove that the corresponding Chevalley group $G(\mathbb{K})$ of adjoint type is isomorphic to $\mathrm{PSL}_n(\mathbb{K})$. This group is defined to be the quotient of $\mathrm{SL}_n(\mathbb{K})$ by its center, which consists of the diagonal matrices with all diagonal entries equal to some nth root of unity in $\mathbb{K}$.