MIDTERM - MATH 2121, FALL 2018.

Name:

Student ID:

Email:

Tutorial: T1A T1B T2A T2B T3A T3B

Problem #	Max points possible	Actual score
1	10	
2	10	
3	20	
4	10	
5	10	
6	20	
Total	80	

You have **120 minutes** to complete this exam.

No books, notes, or electronic devices can be used on the test.

Draw a box around your answers or write your answers in the boxes provided.

Partial credit can be given on some problems if you show your work. Good luck!

Problem 1. (10 points)

Assume h and k are real numbers and consider the linear system

$$x_1 + 3x_2 = k$$
$$4x_1 + hx_2 = 8.$$

Determine all values of h and k such that this system has (i) zero solutions, (ii) a unique solution, or (iii) infinitely many solutions.

Solution:

(i) The system has zero solutions when:

(ii) The system has a unique solution when:

(iii) The system has infinitely solutions when:

Problem 2. (10 points)

Assume A is a 3×3 matrix. The first two columns of A are pivot columns and

$$A \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

What is the reduced echelon form of A?

Solution:

Problem 3. (20 points) Indicate which of the following is TRUE or FALSE.

(1) If a system of linear equations has two different solutions, then it must have infinitely many solutions.
(2) If A is an $m \times n$ matrix and the equation $Ax = b$ is consistent for every b in $\mathbb{R}^m$, then A has m pivot columns.
(3) A linear system with no free variables has a unique solution.
(4) If a linear system $Ax = b$ has more than one solution, then so does the linear system $Ax = 0$.
(5) If $u, v, w \in \mathbb{R}^2$ are all nonzero, then w is a linear combination of u and v.
(6) If A, B, and C are matrices with $AB = AC$, then $B = C$.
(7) If A and B are $m \times n$ matrices, then both AB^T and $A^T B$ are defined.
(8) If A and B are $n \times n$ matrices with $AB = BA$, and if A is invertible, then $A^{-1}B = BA^{-1}$.
(9) If two matrices are row equivalent, then they have the same column space.
(10) If two matrices are row equivalent, then they have the same null space.

Each part will be graded as follows: 0 points for a wrong answer, 1 point for no answer, 2 points for the correct answer. Explanations are not required for answers.

Solution:

(1) TRUE FALSE

(2) TRUE FALSE

(3) TRUE FALSE

(4) TRUE FALSE

(5) TRUE FALSE

(6) TRUE FALSE

(7) TRUE FALSE

(8) TRUE FALSE

(9) TRUE FALSE

(10) TRUE FALSE

Problem 4. (10 points) Let A and B be matrices.

Suppose $AB = \begin{bmatrix} 9 & 8 \\ 7 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$. Find A.

Solution:

Problem 5. (10 points)

(a) Give an example of a 4×3 matrix A such that the linear transformation $T(v) = Av$ is a one-to-one function $\mathbb{R}^3 \to \mathbb{R}^4$. Justify your answer.

(b) Give an example of a 2×3 matrix A such that the linear transformation $T(v) = Av$ is an onto function $\mathbb{R}^3 \to \mathbb{R}^2$. Justify your answer.

Problem 6. (20 points) Consider the matrix

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -13 \end{bmatrix}.$$

Remember that

- The *column space* of A is the span of its columns.
- The *null space* of A is the set of vectors v with $Av = 0$.

(a) Compute the reduced echelon form of A.

(b) Find a basis for the column space of

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -13 \end{bmatrix}.$$

(c) Find a basis for the null space of

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -13 \end{bmatrix}.$$

(d) What are the dimensions of the column space and null space of A?