**Instructions:** Complete the following exercises.

Solutions must be hand-written and submitted in-person.

You will be graded on clarity and simplicity as well as correctness.

You may use any resources and work with other students, but you must write up your own solutions.

Due on **Tuesday, March 17**.

1. Compute the basis of $\mathfrak{sl}_n(\mathbb{F})$ which is dual to the standard basis via the Killing form $\kappa$.

   (This is an upgrade of an earlier exercise which considered the case $n = 2$.)

   Use this to compute $t_\alpha$ and $h_\alpha := 2t_\alpha/\kappa(t_\alpha, t_\alpha)$ for each $\alpha \in \Phi$, relative to the root space decomposition in which $H \subset \mathfrak{sl}_n(\mathbb{F})$ is the maximal toral subalgebra of traceless diagonal matrices.

   Recall that $t_\alpha \in H$ for $\alpha \in H^*$ is the unique element with $\kappa(t_\alpha, h) = \alpha(h)$ for all $h \in H$.

2. Let $L$ be a semisimple Lie algebra with a maximal toral subalgebra $H$.

   Prove that if $h \in H$ then the centralizer $C_L(h) := \{X \in L : [X, h] = 0\}$ is a reductive Lie algebra.

   Prove that there are elements $h \in H$ with $C_L(h) = H$.

   For which $h \in H$ does this hold if $L = \mathfrak{sl}_n(\mathbb{F})$ and $H$ is the subalgebra of traceless diagonal matrices?

3. Assume $L$ is a classical linear Lie algebra of type $C_n$ as defined in the textbook.

   Prove that the set $H$ of all diagonal matrices in $L$ is a maximal toral subalgebra.

4. Assume $L$ is a classical linear Lie algebra of type $C_n$ as defined in the textbook.

   Determine the roots and root spaces corresponding to the root space decomposition of $L$ relative to the maximal toral subalgebra of diagonal matrices $H$.

5. Assume $L$ is a classical linear Lie algebra of type $D_n$ as defined in the textbook.

   Prove that the set $H$ of all diagonal matrices in $L$ is a maximal toral subalgebra.

6. Assume $L$ is a classical linear Lie algebra of type $D_n$ as defined in the textbook.

   Determine the roots and root spaces corresponding to the root space decomposition of $L$ relative to the maximal toral subalgebra of diagonal matrices $H$.