

Math113 – Introduction to Linear Algebra – Homework 1
(Lay’s book; 3rd edition)

§1.1: 22	§1.2: 12, 20
§1.3: 10, 12, 26	§1.4: 33, 38
§1.5: 16, 24	§1.7: 12, 22, 34, 36, 38, 40
§1.8: 6	§1.9: 4, 8, 22

Additional problems

1. True or false questions:

Let $A\mathbf{x} = \mathbf{b}$ be a system of n variables and m equations.

- (a) If $m < n$, then the system is consistent.
- (b) If $m > n$, then the system is inconsistent.
- (c) If the system is consistent and $m = n$, then the system has unique solution.
- (d) If the system is consistent and $m < n$, then the system has infinitely many solutions.
- (e) If the system has unique solution, then $m = n$.

2. True or false questions:

Let $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ be vectors of \mathbb{R}^n .

- (a) If $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ are linearly dependent, so are the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{k-1}$.
- (b) If $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ are linearly independent, so are the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{k-1}$.
- (c) If $k \geq n$, then $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ span \mathbb{R}^n .
- (d) If $k < n$, then $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ cannot span \mathbb{R}^n .
- (e) The vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly dependent if and only if the vectors $\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3, \mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1$ are linearly dependent.

3. Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation, and let $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ be vectors of \mathbb{R}^n .

- (a) If the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ are linearly dependent, so are the image vectors $T(\mathbf{v}_1), T(\mathbf{v}_2), \dots, T(\mathbf{v}_k)$.
- (b) If the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$ are linearly independent, so are the image vectors $T(\mathbf{v}_1), T(\mathbf{v}_2), \dots, T(\mathbf{v}_k)$.
- (c) If the image vectors $T(\mathbf{v}_1), T(\mathbf{v}_2), \dots, T(\mathbf{v}_k)$ are linearly dependent, so are the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$.
- (d) If the image vectors $T(\mathbf{v}_1), T(\mathbf{v}_2), \dots, T(\mathbf{v}_k)$ are linearly independent, so are the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$.

4. Consider the following linear system

$$\begin{cases} x_1 - x_2 + (h-1)x_3 + x_5 = 1 \\ x_1 + (h+2)x_3 - x_5 = 1 \\ +2x_2 + 6x_3 + x_4 = 2 \\ -x_1 + 2x_2 + (4-h)x_3 + x_4 + x_5 = h+4 \end{cases}$$

Determine the values of h such that the system is consistent and find the general solution for those consistent cases.