Math113 – Introduction to Linear Algebra – Homework 2
(Lay's book; 3rd edition)

§2.1: 13, 15, 19   §2.2: 12, 16, 20
§2.3: 9, 18   §2.8: 21, 22, 24
§2.9: 10, 15, 17, 30   §3.1: 10, 13
§3.2: 14, 40   §3.3: 6, 15, 27

Additional problems

1. True or false questions:

   (a) If rows of a matrix are linearly independent, so are the columns.

   (b) If the columns of an n-by-n matrix span $\mathbb{R}^n$, so do the rows.

   (c) If A, B, and C are n-by-n invertible matrices, so is the matrix AB^TC.

   (d) If $AB = AC$ and A is not equal to zero matrix, then $B = C$.

   (e) If $AC = BC$ and C is invertible, then $A = B$.

   (f) Let A and B be same size square matrices. If $\det A = 3$ and $\det B = 5$, then $\det(A + B) = 3 + 5$.

2. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. True or false questions:

   (a) If $n \leq m$, then T is one-to-one.

   (b) If $n \geq m$, then T is onto.

   (c) If $n = m$, then T is one-to-one and onto.

   (d) If T is one-to-one, then $n \leq m$.

   (e) If T is onto, then $n \geq m$.

   (f) If T is one-to-one and onto, then $n = m$.

3. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation defined below. Determine whether T is one-to-one, or onto, or invertible?

   (a) $f(x_1, x_2, x_3) = (x_2 + 7x_3, x_1 + 3x_2 - 2x_3)$.

   (b) $f(x_1, x_2, x_3) = (x_1 + 2x_3, 2x_1 - x_2 + 3x_3, 4x_1 + x_2 + 8x_3)$.

   (c) $f(x_1, x_2, x_3) = (x_1 + x_2 + x_3, x_1 + 2x_2, x_1 + 2x_3)$.

4. Compute $\det A$, $\det A^T$, $\det A^4$, $\det A^{-5}$, A^{-1}, $(A^T)^{-1}$, $(A^{-1})^T$, where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 3 & 6 & 8 \\ 1 & 4 & 8 & 9 \end{bmatrix}.$$

5. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then T is one-to-one if and only if T preserves linear independence (i.e., $T(\boldsymbol{v}_1), \ldots, T(\boldsymbol{v}_k)$ are linearly independent in $\mathbb{R}^m$ whenever $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_k$ are linearly independent in $\mathbb{R}^n$).