Math113 L1: Linear Algebra Midterm Exam (a)

Dept of Math, HKUST, Spring 2004

Name:_____ Tutor:_____

ID No._____ Section:_____

Problem	No.1 (18 pts)	No.2 (15 pts)	No.3 (13 pts)	No.4 (24 pts)	Total (70 pts)
Score					

1. (15 pts) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(x_1, x_2, x_3) = (x_2 + 2x_3, x_1 + 2x_2, 2x_1 + x_3),$$

(a) (3 pts) Find the standard matrix of T.

(b) (5+10 pts) Is T invertible? If so, find the standard matrix of its inverse linear transformation.

2. (15 pts) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation whose standard matrix is $\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 3 & 3 \\ 3 & 3 & 4 & 4 \\ 4 & 4 & 5 & 5 \end{bmatrix}$.

   (a) (10 pts) Find the basic solutions for the homogeneous system

$$T(x_1, x_2, x_3, x_4) = (0, 0, 0, 0).$$

   (b) (5 pts) Find all solutions for the non-homogeneous system

$$T(x_1, x_2, x_3, x_4) = (2, 3, 4, 5).$$

3. (a) (8 pts) Let $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ be the rotation about $\frac{7\pi}{4}$ counterclockwise. Find the standard matrix of T_1.

(b) (5 pts) Let $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection about the straight line $x - y = 0$. Find the standard matrix of T_2.

4. (24 pts)

(a) Let $T : \mathbf{R}^n \to \mathbf{R}^m$ be a linear transformation, and let $v_1, \ldots, v_k$ be vectors of $\mathbf{R}^n$.
(i) If $v_1, \ldots, v_k$ are linearly dependent, then $T(v_1), \ldots, T(v_k)$ are linearly dependent. YES or NO
(ii) If $v_1, \ldots, v_k$ are linearly independent, then $T(v_1), \ldots, T(v_k)$ are linearly independent. YES or NO
(iii) If $T(v_1), \ldots, T(v_k)$ are linearly dependent, then $v_1, \ldots, v_k$ are linearly dependent. YES or NO
(iv) If $T(v_1), \ldots, T(v_k)$ are linearly independent, then $v_1, \ldots, v_k$ are linearly independent. YES or NO

(b) (i) Let v_1, v_2, v_3, v_4 be vectors of $\mathbf{R}^4$ of and $v_4 = 0$. Are v_1, v_2, v_3, v_4 dependent? YES or NO
(ii) Is it possible to find more than eight linearly independent vectors in $\mathbf{R}^5$? YES or NO
(iii) If v_1, v_2, v_3, v_4 are independent, then v_1, v_2, v_3 are independent. YES or NO
(iv) If v_1, v_2, v_3, v_4 are dependent, then v_1, v_2, v_3 are dependent. YES or NO

(c) Let $T : \mathbf{R}^n \to \mathbf{R}^m$ be a linear transformation having the standard matrix A.
(i) T is one-to-one if and only if the number of pivot positions of A is equal to n. YES or NO
(ii) T is one-to-one if and only if the number of pivot positions of A is equal to m. YES or NO
(iii) T is onto if and only if the number of pivot positions of A is equal to m. YES or NO
(iv) T is onto if and only if the number of pivot positions of A is equal to n. YES or NO