

Math2343: Problem Set 2

1. Show that the set \mathbb{Q} of rational numbers is countable.
2. Let Σ be a countable set. Show that Σ^* is countable.
3. Let $B = \{0, 1\}$. Show that $B^\infty := B \times B \times \dots$ is uncountable.
4. Find a one-to-one correspondence between $[0, 1]$ and $(0, 1)$. Find a one-to-one correspondence between the two intervals $[0, 1]$ and $[a, b]$ of real numbers with $a < b$.
5. Define connectives “ \downarrow ” and “ Δ ” in the following table:

p	q	$p \downarrow q$,	p	q	$p \Delta q$
T	T	F		T	T	F
T	F	F		T	F	T
F	T	F		F	T	T
F	F	T		F	F	F

- (a) Find the truth tables for the statements

$$(p \downarrow q) \downarrow r, \quad (p \downarrow q) \wedge (p \downarrow r), \quad (p \downarrow q) \downarrow (p \downarrow r), \quad (p \wedge q) \Delta p, \quad (p \Delta q) \Delta (q \Delta r).$$

- (b) Express $p \downarrow q$ and $p \Delta q$ in terms of p , q , and connectives \neg and \wedge .

6. Let

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- (a) Write the English sentences of the converse, inverse, and contrapositive forms for the statement $p \rightarrow q$.
- (b) Write the English sentences for the statements $\neg p \vee q$ and $\neg q \vee p$ respectively.

7. Show that the statement

$$(\forall x P(x)) \vee (\forall x Q(x)) \rightarrow \forall x (P(x) \vee Q(x))$$

is a tautology. Show that its converse is not a tautology by giving a counterexample.

8. Show that if statements p and $p \rightarrow q$ are tautologies then q is a tautology. Give an example in daily life for the argument.
9. If $p \rightarrow q$ and $\neg q$ are tautologies, then $\neg p$ is a tautology.
10. Determine whether the conclusion follows logically from the premises, and explain why by using propositional calculus.
 - (a) *Premises:* All soldiers can march. Babies are not soldiers.
Conclusion: Babies cannot march.

- (b) *Premises:* (1) If Chan goes to Mainland then Deng is not appointed to Chairman or Fong is appointed to Secretary. (2) Deng is appointed to Chairman if Fong is not appointed to Secretary. (3) Chan goes to Mainland and Deng is not appointed to Chairman.
Conclusion: Fong is appointed to Secretary.