true for all $n \geq 1$ by induction. (As a consequence, if a, b are any two positive integers, then $a = b$, since $P(n)$ is true, where $n = \max(a,b)$.)

First, $P(1)$ is true, since if $\max(a,b) = 1$ then a and b must both be equal to 1. Now assume $P(n)$ true. Let a, b be positive integers such that $\max(a,b) = n+1$. Then $\max(a-1, b-1) = n$. As we are assuming $P(n)$, this implies that $a-1 = b-1$, hence $a = b$. Therefore, $P(n+1)$ is true. By induction, $P(n)$ is true for all n.

There must be something wrong with this "proof." Can you find the error?

13. (a) Suppose we have n straight lines in a plane, and all the lines pass through a single point. Into how many regions do the lines divide the plane? Prove your answer.

(b) We know from Example 8.8 that n straight lines in general position in a plane divide the plane into $\frac{1}{2}(n^2+n+2)$ regions. How many of these regions are infinite and how many are finite?

(In case of any confusion, a finite region is one that has finite area; an infinite region is one that does not.)

14. (See Example 8.9.) Some straight lines are drawn in the plane, forming regions. Show that it is possible to colour each region either red or blue, in such a way that no two neighbouring regions have the same colour.

15. Critic Ivor Smallbrain is sitting through a showing of the new film *Polygon with the Wind*. Ivor is not enjoying the film, and begins to doodle on a piece of paper, drawing circles in such a way that any two of the circles intersect, no two circles touch each other, and no three circles pass through the same point. He notices that after drawing two circles he has divided the plane into four regions, after three there are eight regions, and wonders to himself how many regions there will be after he has drawn n circles. Can you help Ivor?

Chapter 9

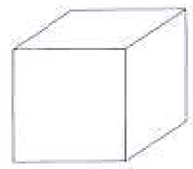
Euler's Formula and Platonic Solids

This chapter contains a rather spectacular proof by induction. The result we shall prove is a famous formula of Euler from the 18th century, concerning the relationship between the numbers of corners, edges and faces of a solid object. As an application of Euler's formula we shall then study the five Platonic solids — the cube, regular tetrahedron, octahedron, icosahedron and dodecahedron.

We shall call our solid objects *polyhedra*. A *polyhedron* is a solid whose surface consists of a number of faces, all of which are polygons, such that any side of a face lies on exactly one other face. The corners of the faces are called the *vertices* of the polyhedron, and their sides are the *edges* of the polyhedron.

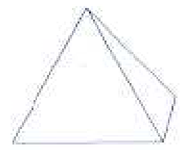
Here are some everyday examples of polyhedra.

(1) *Cube*



This has 8 vertices, 12 edges and 6 faces.

(2) *Tetrahedron*



This has 4 vertices, 6 edges and 4 faces.