Chapter 8

Induction

Consider the following three statements, each involving a general positive integer n:

(1) The sum of the first n odd numbers is equal to n^2.

(2) If $p > -1$ then $(1+p)^n \geq 1+np$.

(3) The sum of the internal angles in an n-sided polygon is $(n-2)\pi$.

(A *polygon* is a closed figure with straight edges, such as a triangle (3 sides), a quadrilateral (4 sides), a pentagon (5 sides), etc.)

We can check that these statements are true for various specific values of n. For instance, (1) is true for $n = 2$ as $1+3 = 4 = 2^2$, and for $n = 3$ as $1+3+5 = 9 = 3^2$; statement (2) is true for $n = 1$ as $1+p \geq 1+p$, and for $n = 2$ as $(1+p)^2 = 1+2p+p^2 \geq 1+2p$; and (3) is true for $n = 3$ as the sum of the angles in a triangle is π, and for $n = 4$ as the sum of the angles in a quadrilateral is 2π.

But how do we go about trying to prove the truth of these statements for *all* values of n?

The answer is that we use the following basic principle. In it we denote by $P(n)$ a statement involving a positive integer n; for example, $P(n)$ could be any of statements (1), (2), or (3) above.

Principle of Mathematical Induction

Suppose that for each positive integer n we have a statement $P(n)$. If we prove the following two things:

(a) *$P(1)$ is true;*

(b) *for all n, if $P(n)$ is true then $P(n+1)$ is also true;*

then $P(n)$ is true for all positive integers n.

The logic behind this principle is clear: by (a), the first statement $P(1)$ is true. By (b) with $n = 1$, we know that $P(1) \Rightarrow P(2)$, hence $P(2)$ is true. By (b) with $n = 2$, $P(2) \Rightarrow P(3)$, hence $P(3)$ is true; and so on.