The principle may look a little strange at first sight, but a few examples should clarify matters.

Example 8.1
Let us try to prove statement (1) above using the Principle of Mathematical Induction. Here $P(n)$ is the statement that the sum of the first n odd numbers is n^2. In other words:

$$P(n) : 1+3+5+\cdots+2n-1=n^2 .$$

We need to carry out parts (a) and (b) of the Principle.
(a) $P(1)$ is true, since $1=1^2$.
(b) Suppose $P(n)$ is true. Then

$$1+3+5+\cdots+2n-1=n^2 .$$

Adding $2n+1$ to both sides gives

$$1+3+5+\cdots+2n-1+2n+1=n^2+2n+1=(n+1)^2 ,$$

which is statement $P(n+1)$. Thus, we have shown that $P(n) \Rightarrow P(n+1)$.
We have now established parts (a) and (b). Hence by the Principle of Mathematical Induction, $P(n)$ is true for all positive integers n.

The phrase "Principle of Mathematical Induction" is quite a mouthful, and we usually use just the single word "induction" instead.

Example 8.2
Now let us prove statement (2) above by induction. Here, for n a positive integer $P(n)$ is the statement

$$P(n) : \text{ if } p>-1 \text{ then } (1+p)^n \geq 1+np .$$

For (a), observe $P(1)$ is true, as $1+p \geq 1+p$.
For (b), suppose $P(n)$ is true, so $(1+p)^n \geq 1+np$. Since $p>-1$, we know that $1+p>0$, so we can multiply both sides of the inequality by $1+p$ (see Example 4.3) to obtain

$$(1+p)^{n+1} \geq (1+np)(1+p)=1+(n+1)p+np^2 .$$

Since $np^2 \geq 0$, this implies that $(1+p)^{n+1} \geq 1+(n+1)p$, which is statement $P(n+1)$. Thus we have shown $P(n) \Rightarrow P(n+1)$.
Therefore, by induction, $P(n)$ is true for all positive integers n.

Next we attempt to prove the statement (3) concerning n-sided polygons. There is a slight problem here. If we naturally enough let $P(n)$ be statement (3),

then $P(n)$ only makes sense if $n \geq 3$; $P(1)$ and $P(2)$ make no sense, as there is no such thing as a 1-sided or 2-sided polygon. To take care of such a situation, we need a slightly modified Principle of Mathematical Induction:

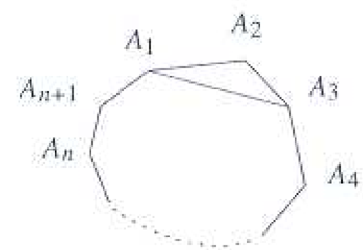
Principle of Mathematical Induction II
Let k be an integer. Suppose that for each integer $n \geq k$ we have a statement $P(n)$. If we prove the following two things:
(a) $P(k)$ is true;
(b) for all $n \geq k$, if $P(n)$ is true then $P(n+1)$ is also true;
then $P(n)$ is true for all integers $n \geq k$.

The logic behind this is the same as explained before.

Example 8.3
Now we prove statement (3). Here we have $k=3$ in the above Principle, and for $n \geq 3$, $P(n)$ is the statement

$$P(n) : \text{ the sum of the internal angles in an } n\text{-sided polygon is } (n-2)\pi.$$

For (a), observe that $P(3)$ is true, since the sum of the angles in a triangle is $\pi=(3-2)\pi$.
Now for (b). Suppose $P(n)$ is true. Consider an $(n+1)$-sided polygon with corners $A_1, A_2, \dots, A_{n+1}$:



Draw the line A_1A_3. Then $A_1A_3A_4 \dots A_{n+1}$ is an n-sided polygon. Since we are assuming $P(n)$ is true, the internal angles in this n-sided polygon add up to $(n-2)\pi$. From the picture we see that the sum of the angles in the $(n+1)$-sided polygon $A_1A_2 \dots A_{n+1}$ is equal to the sum of those in $A_1A_3A_4 \dots A_{n+1}$, plus the sum of those in the triangle $A_1A_2A_3$, hence is

$$(n-2)\pi+\pi=((n+1)-2)\pi .$$

We have now shown that $P(n) \Rightarrow P(n+1)$. Hence by induction, $P(n)$ is true for all $n \geq 3$.