Homework – Hilbert's Axioms

Deadline: Mar. 28, 2013

1. Let A, B, C, D be points a on line l. Prove or disprove by giving a counterexample for the following statements.

   (a) If $A * B * C$ and $B * C * D$, then A, B, C, D are distinct points, and $A * C * D$, $A * B * D$.

   (b) If $A * B * C$ and $A * B * D$, then $A * C * D$ and $B * C * D$.

   (c) If $A * C * D$ and $A * B * D$, then $A * B * C$ and $B * C * D$.

2. **Exterior Angle Theorem** says that the exterior angle of a triangle is larger than its two remote interior angles. Given a triangle ΔABC. Show that $\angle A < \angle B$ if and only if $BC < AC$. (Hint: Applying Exterior Angle Theorem)

3. Let $\mathbb{Q}^2$ be the rational plane of all ordered pairs (x, y) of rational numbers, viewing elements of $\mathbb{Q}^2$ as points and the solution sets of linear equations $ax + by + c = 0$ as lines, where $a, b, c \in \mathbb{Q}$ are fixed constants. Show that Betweenness and Congruence Axioms are satisfied, except Congruence Axiom 1 and Dedekind's Axiom.

4. Show that the interior of a triangle is nonempty.

5. Check if SAS can be replaced by ASA in the congruence axioms.