

**HKUST**

**Mid-term Examination**

**MATH 005 Algebra and Calculus I**

25 October 2000

*Answer ALL questions*

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**Directions** – This is a closed book examination. Work steps must be shown in order to receive full marks. Graphical calculators are not allowed. Note that you can work on *both* sides of the paper and *no* part of these papers is to be torn out. Three sketch papers are attached.

**Student Name:** \_\_\_\_\_

**Student Number:** \_\_\_\_\_

**Instructor Name:** \_\_\_\_\_

**Tutorial Session:** \_\_\_\_\_

Question No.	Marks
1	
2	
3	
4	
Total	

**Question 1**Score: 

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(a) Simplify the following expressions:

(i)  $\sqrt[3]{\frac{27x^3y^6}{125a^9b^3}},$

(ii)  $\frac{x+2y}{2x+y}\sqrt{\frac{2y^2+8xy+8x^2}{2y+x}}$  [5]

(b) Remove the denominators of the following expressions:

(i)  $\frac{3\sqrt{5}+4\sqrt{2}}{3\sqrt{5}-4\sqrt{2}},$

(ii)  $\frac{a+8b}{a^{1/3}+2b^{1/3}}$  [5]

**Question 1 (Cont'd)**

Score:

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(c) Solve the followings equations:

(i)  $x^2 - 2x + 1 - k(x^2 - 1) = 0, (k \neq 1)$   
(Hint: factorization)

[4]

(ii)  $\sqrt{x} + \sqrt{x + \sqrt{1 - x}} = 1.$

[6]

**Question 2**

Score:

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(a) Let  $p = g(q)$  be a quadratic function given by

$$p = g(q) = -q^2 - 4q + 46$$

- (i) Determine the maximum/minimum value of  $p = g(q)$ ; [1]
- (ii) Apply the method of “completing the square” to find the value of  $q$  at which the maximum/minimum value of  $g(q)$  occurs. Find also the value of  $g(q)$  at this  $q$ . [5]
- (iii) Sketch the graph of  $p = g(q)$  [3]

**Question 2 (Cont'd)**

Score:

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(b) Suppose that for a certain commodity the supply and demand curves are given by

$$p = f(q) = q^2 + 2q + 10, \quad (1)$$

and

$$p = g(q) = -q^2 - 4q + 46. \quad (2)$$

However, it is not known which equation represents the supply equation and which equation represents the demand equation.

- (i) Determine the breakeven point; [3]
- (ii) Determine which equations of (1) and (2) are respectively the supply curve and the demand curve by sketching their graphs on the same axes. [7]
- (iii) Give justification to your answers in (ii). [1]

**Question 3**

Score:

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- (a) A worker wants to deposit \$12,000 into a bank for two years. Bank A offers a saving plan that the deposit is compounded quarterly under an annual interest rate 6%, and Bank B offers a saving plan that the deposit is compounded monthly under an annual interest rate 5%. Which plan will yield an higher return?

[7]

- (b) Solve the following equations

$$x + 2y + 3z = 9,$$

$$-4x + y + 6z = -9,$$

$$2x + 7y + 5z = 13$$

simultaneously.

[9]

**Question 4**Score: 





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(a) Let  $g(x)$  be a function defined by

$$g(x) = \begin{cases} |x| - 1, & x \leq 0, \\ x - 1, & 0 \leq x \leq 2, \\ x/2, & 2 \leq x. \end{cases}$$

- (i) Calculate  $g(-3)$ ,  $g(0)$  and  $g(3)$ ; [3]
- (ii) sketch the graph of the function  $g$ ; [4]
- (iii) find the domain of  $g$  (give justification); [2]
- (iv) find the range of  $g$  (give justification). [2]

**Question 4 (Cont'd)**

Score:

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**(b)** Let  $h(x) = 1/\sqrt{x}$ ,  $f(x) = x - 2$  and  $g(x) = \log x$ . Find the expressions and domains of the following functions:

(i)  $h(h(x))$ ,    (ii)  $(h(f(x)))^3$ ,    (iii)  $f(h(x) + e^{\frac{1}{2}g(x)})$ . [9]

**(c)** Find the inverse of  $y = f(x) = x^2 + 4x - 12$ .

[4]







