HKUST MATH 005

Final Examination
Algebra and Calculus I

19 December 2001; 8:30-11:30

Answer ALL questions; Full mark = 100

Time allowed – 3 hours

Directions – This is a closed book examination. Work steps may help to gain partial credits.

Question 1. (a) Compute the following limits (Give an explanation if you consider the limit does not exist; you may use the symbols $+\infty$ and $-\infty$ as appropriate.)

(i) $\displaystyle\lim_{x\to -1} \frac{2x^2 - x - 6}{x^2 - 4}$, [2]

(ii) $\displaystyle\lim_{x\to -\infty} \frac{e^{x-2} - 4}{5}$. [2]

(iii) $\displaystyle\lim_{q\to -\infty} \frac{7q^2 + 3q + 1}{2q + 3}$, [2]

(b) $\displaystyle\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$, where $f(x) = 2x^2 + 3x + 5$. [3]

(c) Suppose

$$f(x) = \begin{cases} x+2, & \text{if } x > 0, \\ x^2 + 3, & \text{if } x \le 0. \end{cases}$$

Find $\lim_{x\to 0} f(x)$. [3]

Question 2 (a) Compute the first derivative of the following expressions.

(i) $y = \sqrt[4]{(2x+5)^3}$. [2]

(ii) $y = \ln\big(\ln(2x+1)\big)$, [2]

(iii) $y = x^{3x+1}$. [2]

(b) Find the value of x for which the curve $y = 6x^2 + 4x - 5$ has slope 16. Then find the equation of the tangent passing through such point. [3]

Question 3 (a) Solve the equation $2^{2x} - 2(2^x) - 3 = 0$. [3]

(b) Find all those numbers x such that

$$\frac{x+5}{x^2 + 2x - 8} \ge 0.$$

[4]

(c) Solve the following system of equations. [4]

$$\begin{aligned} x + 2y + 3z &= 1 \\ -2x + 2y + z &= 1 \\ 0x + y + z &= 1. \end{aligned}$$

Question 4 (a) Perform the following indefinite integrations.

(i) $\displaystyle\int \frac{1}{\sqrt[4]{3x+2}}\,dx$, [2]

(ii) $\displaystyle\int \frac{1}{\sqrt{2x+1}} e^{\sqrt{2x+1}}\,dx$. [2]

(b) Compute the following definite integrations.

(i) $\displaystyle\int_{\sqrt{2}}^{2} \frac{3x}{x^2-1}\,dx,$ [3]

(ii) $\displaystyle\int_{0}^{1} (x+4)\sqrt[3]{x^2+8x-1}\,dx.$ [3]

Question 5 (a) Find the present values of \$ 71,000 to be paid 15 years later if we assume the interest rate is 8% compounded

(i) monthly, [2]

and (ii) continuously. [2]

(b) Suppose a corporation pays \$50,000 for a machine that has a useful life of eight years and a salvage value of \$5,000. A sinking fund is established to replace the machine at the end of 8 years. The replacement machine will cost \$70,000. If equal payments are made into the fund at the end of every 6 months and the fund earns interest at the rate 10% compounded semiannually, what should each payment be? [6]

Question 6 The demand function for a manufacturer's product is given by

$$p = \frac{400}{q+2},$$

where p is the price per unit when q units are demanded. (You may assume the elasticity of demand is given by $\eta = \frac{dq/q}{dp/p} = \frac{p/q}{dp/dq}$.)

(a) Find the point elasticity of demand function. [2]

(b) Determine the range of q for which the point elasticity is

(i) elastic, [2]

(ii) inelastic, [2]

and (iii) unit elasticity. [2]

(c) Find the point elasticity of demand when $q = 100$ and interpret your result. [2]

Question 7 (a) A company owns an apartment building containing 100 units. If the company charges a rent of \$400 per month, then all units can be rented out and for every increase of \$ 20 per month in rent, the company will lose one customer. What rent should be charged to maximize revenue? [6]

(b) (i) From the fact that the cube root of 8 is equal to 2 use differentials to give an approximation to the number $\sqrt[3]{8.2}$. (Note that you must show all the steps in using the differential. A simple numerical answer without proper explanation will not be accepted.) [3]

(ii) Explain how accurate your answer obtained in (i) is by comparing it with the answer that you obtained by using a calculator. [1]

Question 8 Suppose the demand equation for a certain product is $p = 200 - q^2$, where p is the price per unit for q units, and the supply equation is $p = 6q + 160$.

(i) Find the intersection of the supply and demand curves. Sketch the curves. [2]

(ii) Find the consumers' surplus when market reaches its equilibrium. [3]

(iii) Find the producers' surplus when market reaches its equilibrium. [3]

Question 9 Let $y = f(x)$ be a function of x given by

$$f(x) = (x^2 - 3x + 2)^2.$$

(i) Write down the x-coordinate(s) for which $f(x) = 0$. [2]

(ii) Evaluate the limits $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to+\infty} f(x)$. [2]

(iii) Find the critical point(s) of $f(x)$. [5]

(iv) Determine the region(s) on the x-axis where f is increasing and decreasing respectively. [4]

(v) Determine whether each of the critical point(s) found in (iii) is a relative maximum or minimum. [3]

(vi) Sketch the graph of $f(x)$. [4]