HKUST

First Mid-term Test

MATH 005 Algebra and Calculus I

4 October 2001

Answer ALL questions

Full mark: 90; each question may carry different mark.

Time allowed – 1 hour 30 minutes

Directions – This is a closed book examination. Work steps must be shown in order to receive full marks. No laptop computers are allowed. Note that you can work on *both* sides of the paper.

Student Name: ____________________________

Student Number: ____________________________

Instructor Name: ____________________________

Tutorial Session: ____________________________

Question No. (mark)	Marks
1 (20)	
2 (10)	
3 (12)	
4 (10)	
5 (9)	
6 (21)	
7 (8)	
Total	

Question 1 Score: ______

(a) Simplify the following expressions. Please show your computations.

(i) $\left(\dfrac{1}{32}\right)^{-2/5}$, (ii) $\sqrt[5]{(64)^{5/6}}$.

[4]

(b) Simplify the following expressions and leave your answers without negative exponents (indices).

(i) $(16x^{-2})^{5/4}$, (ii) $\left(\dfrac{3x^{-4}y}{z^2}\right)^{-3}$. [4]

(c) Multiply out the following expressions.

(i) $(x+1)(x-1)(x^2-1)$, (ii) $(2x+3)^3$. [4]

(d) Factorize completely the following expressions.

(i) $(x+4)^2-(4-x)^2$, (ii) $a^4-8a^2b^2-9b^4$, [4]

(e) Solve the following inequalities.

(i) $\dfrac{2x-1}{-2} < \dfrac{4x-3}{2}$, (ii) $|2x-9| < 3$. [4]

Question 2 Score: ☐

(a) Combine and simplify the followings expressions.

(i) $\dfrac{3}{x(x-1)} - \dfrac{x-1}{x^2(x+1)}$, [3]

(ii) $\dfrac{x}{x^2-2x-8} - \dfrac{2}{x^2+4x+4} + \dfrac{3}{x-4}$. [4]

(b) Perform the following long division.

$(x^3 + 3x^2 + 5x + 7) \div (2x^2 - 1)$. [3]

Question 3 Score: [ ]

(a) Solve the following equations.

(i) $\sqrt{2x+3} - \sqrt{2x-3} = 1$, [4]

(ii) $\dfrac{6}{x^2-1} = \dfrac{3}{x+1} + 1$. [4]

(b) Given

$$\frac{1}{2}x - \frac{1}{4}y = \frac{1}{6},$$
$$x + \frac{1}{2}y = \frac{2}{3}.$$

(i) Solve the above system of equations, [2]

(ii) Sketch the graph of the above equations. [2]

Question 4 Score:

Suppose that the demand function for a manufacturer's product is $p = p(q) = 800 - 2q$, where p is the price in dollars per unit where q units are demanded per week.

(i) The revenue function is equal to the product of price and quantity. Write down the revenue as a function of q. [1]

(ii) Use the method of completing the square to determine the quantity q that maximizes the revenue. [4]

(iii) Hence find the maximum revenue generated by the revenue function in (i), [1]

(iv) and sketch the graph of the revenue function in (i). [4]

Question 5 Score:

(a) A person wishes to deposit a total of \$ 10,000 into two accounts. The saving account pays yearly interest 4% and the fixed certificate of deposit pays a yearly interest rate of 7 %. How much should the person deposit in each account, so that he/she gets a total of \$ 502 interest at the end of the year? [5]

(b) An elder sister is currently twice as old as her younger brother. Six years ago the elder sister was three times as old as the younger brother. Find the present age of the sister and the brother. [4]

Question 6 Score: ☐

(a) Let $f(x)$ be the function defined by

$$f(x) = \begin{cases} 2x, & \text{if } 0 < x < 1, \\ 1 - x, & \text{if } 1 \le x \le 2, \\ 0, & \text{if } 2 < x \le 3. \end{cases}$$

(i) Find the domain of $f(x)$. [1]

(ii) Write down the values of $f(1/2)$, $f(1)$, $f(\sqrt{2})$ and $f(4)$. [4]

(iii) Sketch the graph of the function $f(x)$. [4]

(iv) Find the range of f. [2]

(b) Let $f(x) = x^2 - 1$, and $g(x) = \sqrt{x}$.

(i) Find the domains of $f(x)$ and $g(x)$. [3]

(ii) Find $f(g(x))$ and the values of $f(g(4))$, $f(g(-2))$. [3]

(iii) Find $g(f(x))$ and its domain. [4]

Question 7 Score:

(a) Solve the equation $\log_2(x+4) = 3$. [4]

(b) A radioactive substance decays according to the equation $N = N(t) = 10e^{-0.04t}$, where N is the number of milligrams present after t days.

(i) Find the initial value of N (that is, when $t = 0$). [1]

(ii) Find the time T needed so that the initial amount of the radioactive substance is decayed by half. [3]