# HKUST

## Second Mid-term Test

# MATH 005 Algebra and Calculus I

8 November 2001

*Answer ALL questions*

*Full mark: 60; each question may carry different mark.*

*Time allowed – 1 hour 30 minutes*

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**Directions** – This is a closed book examination. Work steps must be shown in order to receive full marks. No laptop computers are allowed. Note that you can work on *both* sides of the paper.

**Student Name:** _____

**Student Number:** _____

**Instructor Name:** _____

**Tutorial Session:** _____

| **Question No.** (mark) | Marks |
|---|---|
| **1** (8) | |
| **2** (13) | |
| **3** (6) | |
| **4** (9) | |
| **5** (14) | |
| **6** (10) | |

**Question 1** Score: ☐

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**(a)** Evaluate the following limits.

(i) $\lim_{x \to +\infty} \dfrac{21x^2 - 6x + 1}{3x^2 + 2x - 1}$. [2]

(ii) $\lim_{n \to +\infty} \dfrac{2n + 3 - 4n^2}{n^3 + n - 1}$. [2]

**(b)** Let $f(x) = \begin{cases} 3 - x, & \text{if } x > 2 \\ 3x - 5, & \text{if } x < 2. \end{cases}$

(i) Sketch the graph of $f(x)$, [2]

(ii) Find $\lim_{x \to 2} f(x)$. [2]

**Question 2** Score:

**(a)** Differentiate the following functions.

(i) $f(x) = 4x^3 - 6x^2 + 7x - 8$, [2]

(ii) $y = \dfrac{x^2 + 2x + 3}{\sqrt{x}}$. [2]

(iii) $y = x^4(3x - 1)^{15}$. [3]

(iv) $y = \dfrac{\sqrt{x}}{2x^2 + 1}$. [3]

**(b)** Suppose $y = u^5 - 8u^2 + 2u - 1$, $u = \sqrt{x + 10}$. Find $\dfrac{dy}{dx}$ when $x = -9$. [3]

**Question 3** Score: ☐

Let $f(x) = 4x^2 - 3x + 1$.

(i) Find $f'(x)$. [2]

(ii) Evaluate $f'(1)$. [1]

(iii) Find the equation of the tangent line to the graph of $y = f(x)$ at $x = 1$. [3]

**Question 4** Score:

**(a)** Let $f(x) = ax^2 - bx$, where $a$ and $b$ are constants. Find $f'(x)$ by computing the limit:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

[4]

**(b)** Suppose a manufacturer's average cost is given by

$$\overline{c} = 0.001q^2 - 0.12q + 7 + \frac{7000}{q},$$

where $q$ denotes the quantity of a commodity, $\overline{c} = c(q)/q$, and $c(q)$ is the cost function of the manufacturer's cost.

(i) Find the marginal cost function; [3]

(ii) Evaluate the marginal cost when 50 units are produced. [1]

(iii) Give an economic interpretation of your answer to (ii). [1]

**Question 5** Score:

Let $f(x) = 2x^3 + 3x^2 - 36x + 1$ be a function of $x$.

(i) Find all the critical points of $f(x)$ (that is the $x$ for which $f'(x) = 0$). [4]

(ii) Determine the intervals of the $x$-axis on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing. [6]

(iii) Use the first derivative test to determine whether each of the critical points of $f(x)$ found in (i) above is a relative maximum or a relative minimum or neither of $f(x)$. [4]

**Question 6** Score:

**(a)** A debt which is equal to $ 2,000 paid one year from now, is to be repaid by a payment of $ 1,000 three years from now and a payment two years from now. If the interest rate is 4% compounded annually, what amount is to be paid two years from now? (Show your steps in order to receive full marks.)

[4]

**(b)** Consider the following annuity: $2,000 due at the end of each year for two years, and $ 3,000 due thereafter at the end of each year for three years. Find the present value of the annuity if the interest is compounded at an annual rate of 7 %.

[6]