

HKUST

MATH005 ALGEBRA AND CALCULUS I

First Mid-Term Examination (Version A)

Name: _____

8th October 2002

Student I.D.: _____

19:00–20:30

Tutorial Section: _____

Directions:

- Do NOT open the exam until instructed to do so.
- Please write your name, ID number, and Section in the space provided above.
- Answer ALL questions.
- This is a closed book examination.
- No graphical calculators are allowed.
- You may write on both sides of the examination papers.
- Once you are allowed to open the exam, please check that you have 7 pages of questions in addition to the cover page.
- You must show the working steps of your answers in order to receive full marks.
- All mobile phones and pagers should be switched off during the examination.
- Cheating is a serious offense. Students who commit this offense may receive zero mark in the examination. However, more serious penalty may be imposed.

Question No.	Q. 1	Q. 2	Q. 3	Q. 4	Q. 5	Q. 6	Q. 7	Q. 8	Q. 9	Q. 10
Points (4 each)										

Question No.	Points	Out of
Q. 11		15
Q. 12		13
Q. 13		16
Q. 14		16
Q. 1-10		40
Total Points		100

Part I: Answer each of the following 10 multiple choice questions.
Each is worth 4 points. No partial credit.

- If $f(x) = g(x + 3)$ and $g(x) = x^2 + 2x + 1$, compute $f(1)$.
 (a) 24 (b) 25 (c) 16 (d) 17 (e) None of the previous

- Jenny has decided to pay off two loans on which she has not made any previous repayment. The first loan is a loan in which she obtained \$9,000 two years ago at an annual interest rate of 8% compounded semiannually. The second is a loan where \$12,500 is due two years from now at an annual interest rate of 16% compounded quarterly. To the nearest dollar, how much is due now?
 (a) \$23,391 (b) \$25,235 (c) \$22,032 (d) \$19,662 (e) \$22,378

- The domain of the function defined by the formula $f(x) = \frac{x^2 - 1}{2x^3}$ is:
 (a) All real numbers except 0
 (b) All real numbers except 1
 (c) All real numbers except ± 1
 (d) All real numbers > 0
 (e) All real numbers except $1/2$

- The derivative of $f(x) = \frac{x^2 + 1}{2} - \frac{1}{x^{\frac{3}{2}}}$ is:
 (a) $\frac{2x + 1}{2} + \frac{3}{2x^{\frac{5}{2}}}$ (b) $x - \frac{3}{2x^{\frac{5}{2}}}$ (c) $x + \frac{3}{2x^{\frac{5}{2}}}$
 (d) $\frac{2x + 1}{2} - \frac{3}{2x^{\frac{1}{2}}}$ (e) $x - \frac{3}{2x^{\frac{1}{2}}}$

5. Find the limit $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^2 + 2x - 3}$

- (a) -1 (b) 1 (c) $-\frac{1}{3}$ (d) 0 (e) Does not exist

6. Determine the tangent line to the graph of the function $y = f(x) = -6x^2 + 3x - 2$ at the point $(2, f(2))$.

- (a) $y = -21x + 22$ (b) $y = 23x - 22$ (c) $y = 12x + 4$
 (d) $y = 23x + 22$ (e) $y = 4x - 1$

7. Compute $g'(4)$ for $g(x) = (31 - 15\sqrt{x})(x^2 - 16)$.

- (a) -8 (b) 263 (c) 24 (d) 0 (e) 8

8. Conversion between temperature measured in Fahrenheit F and measured in Celsius C is a linear function. Water freezes at 32F and 0C, and boils at 212F and 100C. The temperature in New York City is 68F. What is this temperature in C?

- (a) 20 (b) $-\frac{160}{9}$ (c) $\frac{230}{9}$ (d) -10 (e) 30

9. Find the maximum/minimum of the function $y = f(x) = 2x^2 + 5x - 3$.

- (a) $y = -\frac{49}{8}$, min. (b) $y = -\frac{5}{4}$, min. (c) $y = -\frac{5}{4}$, max. (d) $y = -\frac{49}{8}$, max. (e) $y = \frac{13}{4}$, max.

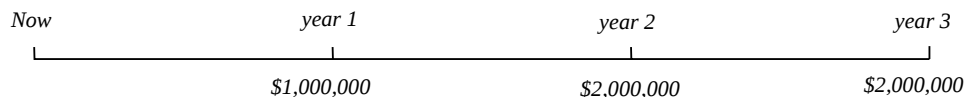
10. A vertical pole of height 4 metres snaps at a height of x metres above the ground. The upright portion (of x -metres) now forms the *perpendicular* side of a right-angled triangle, the snapped upper portion forms the *hypoteneuse* of the right-angled triangle (with the *base* of the triangle being along the ground). Determine the function $f(x)$ that gives the area (in square metres) of the relevant right-angled triangle.

- (a) $f(x) = \frac{xH}{2}$ (b) $f(x) = \frac{x(4-x)}{2}$ (c) $f(x) = \frac{x}{2}\sqrt{16-8x}$
 (d) $f(x) = \frac{4-x}{2}\sqrt{4x-16}$ (e) $f(x) = \frac{x}{2}\sqrt{16-x^2}$

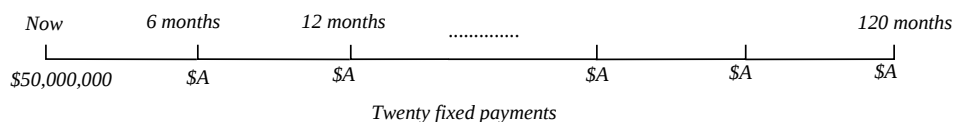
Part II: Answer each of the following 4 long questions.

11. For both parts (a) and (b), assume that the interest rate is 8% per year compounded every 6 months. Show your calculations and write down any formula you use.

- (a) Company C has just received a loan and will repay it with three payments. The first payment, due one year from now, will be \$1,000,000. The second and third payments, due two and three years from now, will be \$2,000,000 each. Determine the loan amount. (7 points)



- (b) Company B has a debt of \$50,000,000 and wishes to repay it with twenty equal payments of \$A, with the first payment due in 6 months and then every 6 months afterwards. Determine A. (8 points)



12. Maggie's Candy Store makes chocolate at a cost of \$15/kg, and sells it at a price of \$30/kg. Customers have been consuming 500 kgs of chocolate each day at the \$30/kg price. Maggie wishes to increase her profit by raising her chocolate price, but estimates that for each \$1 increase in the price, 20 less kgs will be sold each day. Let x be the selling price (in \$/kg) of Maggie's chocolate.

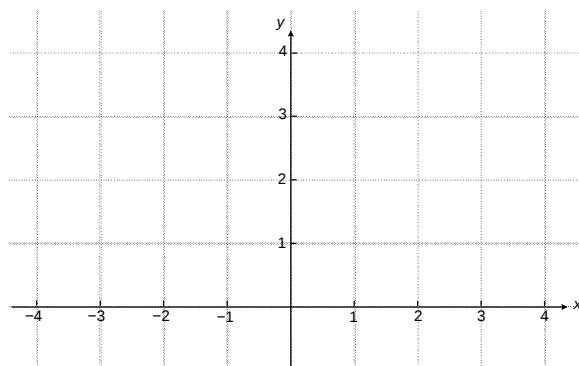
(a) Express the amount of chocolate purchased each day as a linear function of x . (2 points)

(b) Express Maggie's profit as a function of x . (4 points)

(c) Determine the price x that yields Maggie's maximum profit. What is the maximum profit? Explain your reasoning. (7 points)

13. Consider the functions $f(x) = \frac{1}{x^2}$ and $g(x) = |2x|$.

- (a) Sketch the graphs of $f(x)$ and $g(x)$ using the coordinate axes below, and indicate accurately the coordinates of at least three points on each of the graphs you sketch. (5 points)

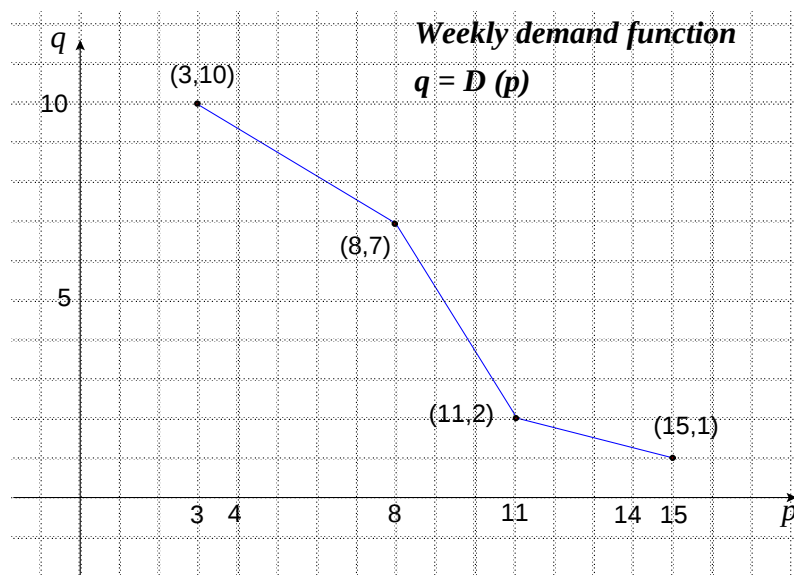


- (b) State the limit definition of the derivative of a function at a point x . (3 points)

- (c) **Using the limit definition**, find the derivative of $f(x) = \frac{1}{x^2}$. (5 points)

- (d) Can the above functions $f(x)$ and $g(x)$ have the same rate of change (i.e., derivative) at some value of x ? Explain your reasoning. (1 + 2 points)

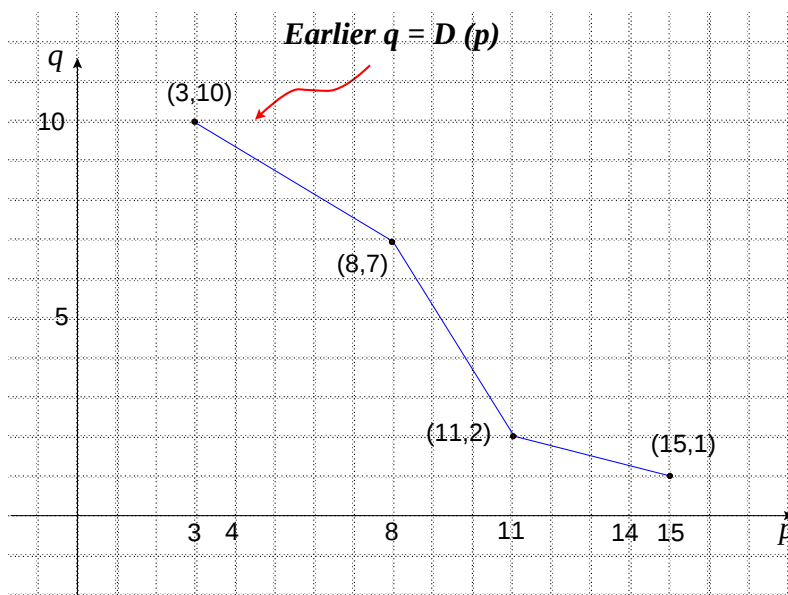
14. The figure below is the graph of a piecewise linear function, which is used to model the weekly demand $q = D(p)$ of ground beef sold in BBQ Supermarket. The variable p is the price (in \$/kg) and q is the quantity (in kgs) of beef sold.



- (a) Write down the explicit formula for the demand function $q = D(p)$, on each of the linear pieces shown in the above figure. (7 points)

- (b) At what price(s) p within the interval $4 < p < 14$ will the derivative of the demand function not exist? Explain your reasoning. (4 points)

- (c) The outburst of the mad cow disease has changed the weekly demand function. People are now paying two dollars less for the same amount of beef than in the old days. Sketch the graph of the new weekly demand function $q = D_{\text{new}}(p)$ in the figure below. (3 points)



- (d) Express $q = D_{\text{new}}(p)$ in terms of $D(p)$. (2 points)