MATH1013 Calculus I

Introduction to Functions[1]

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Integration II (Chapter 5)

[1] Based on Briggs, Cochran and Gillett: Calculus for Scientists and Engineers: Early Transcendentals, Pearson 2013

A Simple Inequality

It follows from the definition of the definite integral that if f and $|f|$ are both integrable over $[a, b]$ then

$$\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx,$$

and in particular if f attains its maximum M at some point inside $[a, b]$, then $\int_a^b f(x)\,dx \leq M(b-a)$.
This follows easily from the $|a+b| \leq |a| + |b|$ for any a, b

$$\begin{aligned}\left|\int_a^b f(x)\,dx\right| &= \left|\lim_{\Delta\to 0}\sum_{k=1}^{n} f(x_k^*)\,\Delta x_k\right| \\ &\leq \lim_{\Delta\to 0}\sum_{k=1}^{n} |f(x_k^*)|\,\Delta x_k = \int_a^b |f(x)|\,dx,\end{aligned}$$

as required. Moreover, since $|f(x)| \leq M$ for all x, so the last inequality also follows.

Fundamental theorem of calculus (Issac Newton)

Theorem 5.3 *Suppose that f is continuous over $[a, b]$ and that f has a primitive (antiderivative) $F(x)$ over $[a, b]$, that is, $F'(x) = f(x)$ on $[a, b]$. Then*

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a).$$

Example Find $\int_2^3 x^2\,dx$ by Newton's Fundamental theorem of calculus.
Since $F(x) = \frac{1}{3}x^3$ is a primitive of $f(x) = x^2$, so

$$\int_2^3 x^2\,dx = F(3) - F(2) = \frac{1}{3}\,3^3 - \frac{1}{3}2^3 = \frac{19}{3}.$$

Examples of Fundamental theorem of calculus

- $$\int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}(1^2 - 0^2) = \frac{1}{2}. \quad (F(x) = x^2/2)$$

- $$\int_{-1}^3 3x^2 - x + 6\,dx = x^3 - \frac{1}{2}x^2 + 6x\Big|_{-1}^3 = 48.$$

- $$\int_a^b 2x - 5x^2\,dx = x^2 - \frac{5}{3}x^3\Big|_a^b = (b^2 - a^2) + \frac{5}{3}(a^3 - b^3).$$

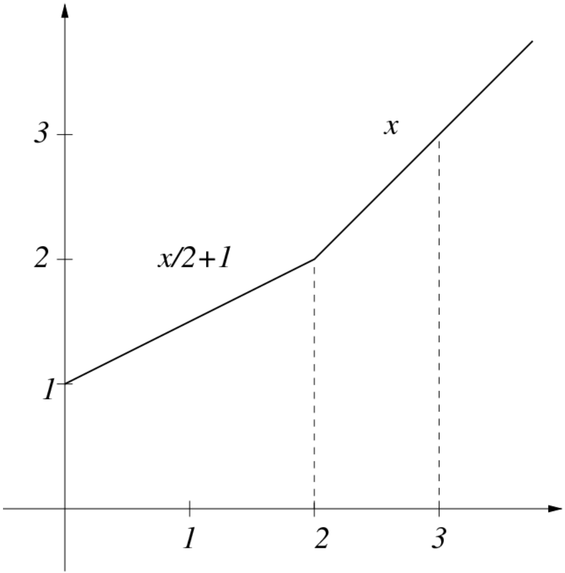
A piecewise continuous function

Example Find the area under the following function over $[0, 3]$

$$f(x) = \begin{cases} \frac{x}{2} + 1, & \text{if } 0 \le x \le 2; \\ x, & \text{if } 2 < x \le 3. \end{cases}$$

$$\begin{aligned} \int_0^3 f(x)\,dx &= \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx \\ &= \int_0^2 \left(\frac{x}{2} + 1\right)\,dx + \int_2^3 x\,dx \\ &= \frac{1}{2}\int_0^2 x\,dx + \int_0^2 1\,dx + \int_2^3 x\,dx \\ &= \frac{1}{4}x^2\Big|_0^2 + x\Big|_0^2 + x^2/2\Big|_2^3 \\ &= 11/2. \end{aligned}$$

Figure of the piecewise continuous function



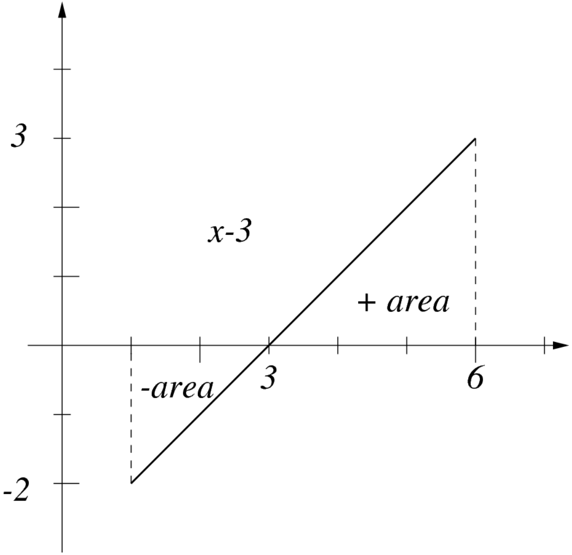
An example with net area

Example Find the area under $f(x) = x - 3$ over $[1, 6]$. It is clear that f is negative over $[1, 3]$.

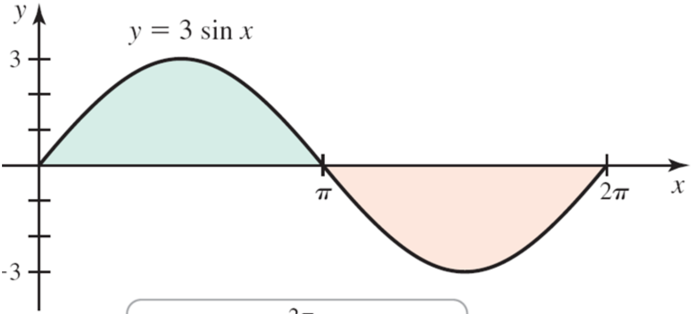
$$\begin{aligned} Area &= \left| \int_1^3 x - 3\, dx \right| + \int_3^6 x - 3\, dx \\ &= \left| x^2/2 - 3x \Big|_1^3 \right| + (x^2/2 - 3x)\Big|_3^6 \\ &= |(9/2 - 9) - (1/2 - 3)| + (9^2/2 - 18) - (3^2/2 - 9) \\ &= |-2| + 9/2 = 13/2. \end{aligned}$$

Whereas the net area after taking away the negative area, $\int_1^6 (x - 3)\, dx = 5/2$.

An example with net area



Second example with net area (a)



$$\text{Net area} = \int_0^{2\pi} 3 \sin x \, dx = 0.$$

Second example with net area (b)

It follows from Newton's second fundamental theorem of calculus that

$$\int_0^{2\pi} 3\sin x\, dx = -3\cos x\Big|_0^{2\pi}$$
$$= -3\cos(2\pi) - (-3\cos 0) = -3(1) + 3(1) = 0.$$

Proof: Primitive as area function

Let

$$A(x) = \int_a^x f(z)\,dz$$

be the function which gives the area under f from a to x. Then

$$\begin{aligned}
&\left|\frac{1}{h}(A(c+h) - A(c)) - f(c)\right| \\
&= \left|\frac{1}{h}\left(\int_a^{c+h} f(x)\,dx - \int_a^{c} f(x)\,dx\right) - f(c)\right| \\
&= \left|\frac{1}{h}\int_c^{c+h} f(x)\,dx - \frac{1}{h}\int_c^{c+h} f(c)\,dx\right| \\
&\leq \frac{1}{h}\int_c^{c+h} \left|f(x) - f(c)\right| dx \leq \max_{c\leq x\leq c+h}\left(\left|f(x) - f(c)\right|\right)\frac{1}{h}\int_c^{c+h} 1\,dx \\
&= \max_{c\leq x\leq c+h}\left|f(x) - f(c)\right| \to 0,
\end{aligned}$$

as $x \to c$ since f is continuous at c.

Completion of proof

What we have shown is that

$$\frac{1}{h}\left(A(c+h)-A(c)\right) \to A'(c) = f(c), \quad h \to 0.$$

And the above applies at all points in $[a, b]$ (except at the end points a and b which require some modifications). Hence the function $A(x)$ is a primitive of f. Thus

$$A(x) = F(x) + C \quad \text{for some constant } C.$$

Since $0 = A(a) = F(a) + C$. So $C = -F(a)$. But then at $x = b$,

$$F(b) + C = A(b) = \int_a^b f(x)\,dx.$$

Hence $\int_a^b f(x)\,dx = F(b) - F(a)$ and this completes the proof.

Area function figure 1

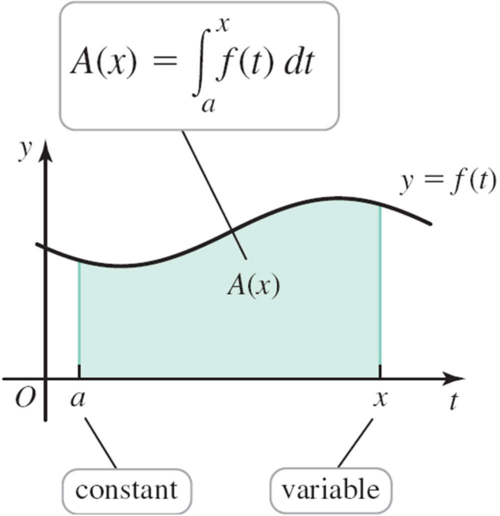


Figure: (Publisher Figure 5.32)

Area function figure 2

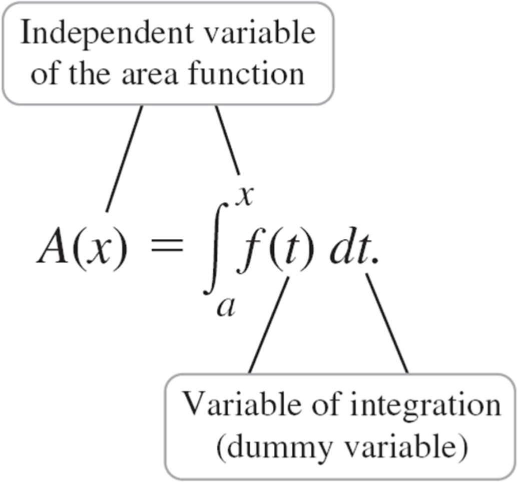


Figure: (Publisher Figure 5.32a)

Area function figure 3

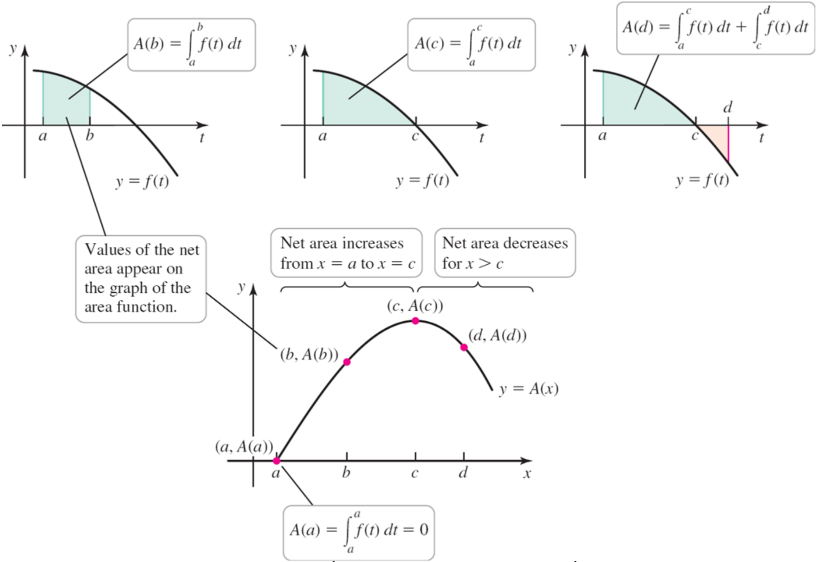


Figure: (Publisher Figure 5.33)

Area function figure 4

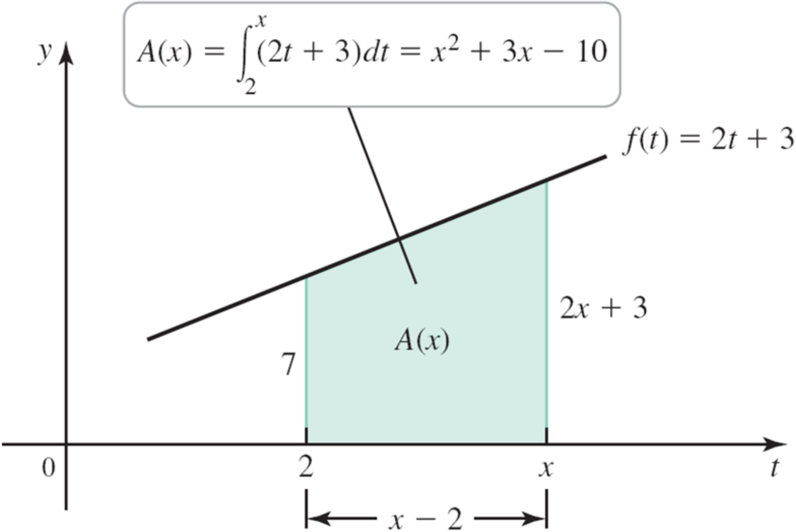


Figure: (Publisher Figure 5.37)

Area function figure 5

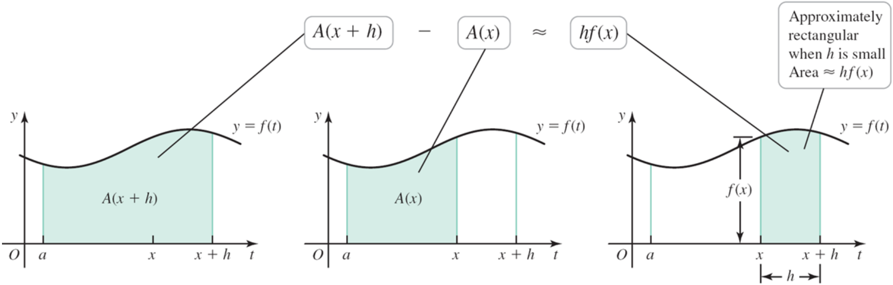


Figure: (Publisher Figure 5.39)

Area function figure 6

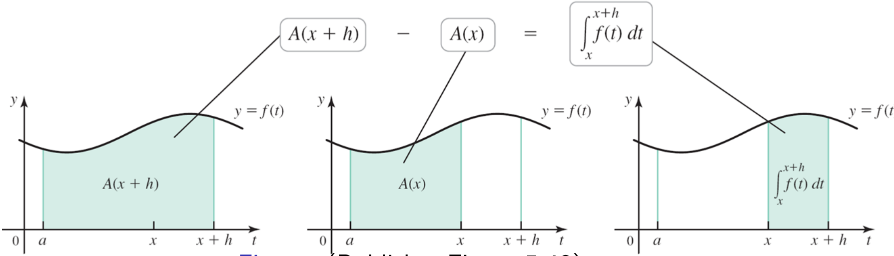


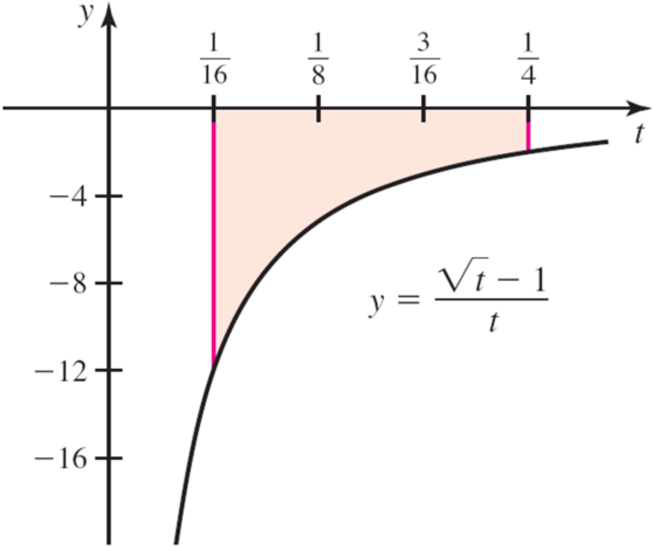
Figure: (Publisher Figure 5.48)

Example on dummy variable (pp. 361-362)

We may use other variable such as t instead of x in definite integration:

$$\begin{aligned}
\int_{1/16}^{1/4} \frac{\sqrt{t}-1}{t}\,dt &= \int_{1/16}^{1/4} \left(t^{-1/2} - \frac{1}{t}\right) dt \\
&= 2\,t^{1/2} - \ln|t| \Big|_{1/16}^{1/4} \\
&= \left[2\left(\frac{1}{4}\right)^{1/2} - \ln\frac{1}{4}\right] - \left[2\left(\frac{1}{16}\right)^{1/2} - \ln\frac{1}{16}\right] \\
&= 1 - \frac{1}{2} + \ln\frac{1}{16} - \ln\frac{1}{4} \\
&= \frac{1}{2} + \ln\frac{1}{4} = \frac{1}{2} - \ln 4 \\
&\approx 0.5 - 1.38263 = -0.8863
\end{aligned}$$

Figure on dummy variable (Figure 5.43)



Example on Derivatives of integral (pp. 362-363)

Given $f(x)$ on $[a, b]$, we recall from the proof of the Fundamental theorem of calculus that the area function defined by

$$F(x) = \int_a^x f(t)\, dt$$

serves as a primitive of f. That is, $F'(x) = f(x)$.
So we have

(1)

$$\frac{d}{dx}\left(\int_1^x \sin^2 t\, dt\right) = \sin^2 x.$$

More example on Derivatives of integral (pp. 362-363)

(2)

$$\frac{d}{dx}\left(\int_x^5 \sqrt{t^2+1}\,dt\right) = \frac{d}{dx}\left(-\int_5^x \sqrt{t^2+1}\right)dt$$
$$= -\frac{d}{dx}\left(\int_5^x \sqrt{t^2+1}\,dt\right) = \sqrt{x^2+1}.$$

(3) We need to apply Chain rule to

$$\frac{d}{dx}\left(\int_0^{x^2} \cos t^2\,dt\right) = \frac{d}{du}\left(\int_0^{u} \cos t^2\,dt\right)\cdot\frac{du}{dt}, \qquad \text{where } u = x^2$$
$$= \cos u^2 \cdot (2x)$$
$$= 2x\cos(x^2)^2 = 2x\cos x^4.$$

Exercises (p. 367)

1. $\frac{d}{dx}\left(\int_3^x t^2 + t + 1\, dt\right)$
2. $\frac{d}{dx}\left(\int_2^x \frac{dp}{p^2}\right)$
3. $\frac{d}{dx}\left(\int_0^x e^t\, dt\right)$
4. $\frac{d}{dx}\left(\int_2^{x^2} \frac{dp}{p^2}\right)$
5. $\frac{d}{dx}\left(\int_x^1 \sqrt{t^4 + 1}\, dt\right)$
6. $\frac{d}{dx}\left(\int_{x^2}^{10} \frac{dz}{z^2 + 1}\right)$

Even functions

We call a function even if

$$f(-x) = f(x), \qquad \text{for all } x$$

That is, we have

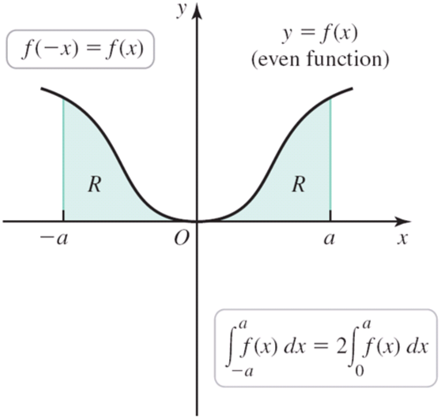


Figure: (Publisher Figure 5.50a)

Odd functions

We call a function even if

$$f(-x) = -f(x), \qquad \text{for all } x$$

That is, we have

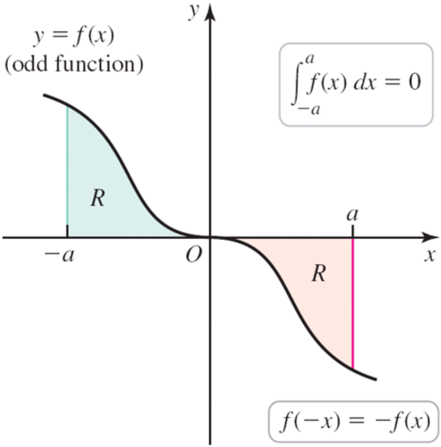


Figure: (Publisher Figure 5.50b)

Even and Odd examples

- $$\int_{-2}^{2}(x^4 - 3x^3)\,dx = \int_{-2}^{2} x^4\,dx - 3\int_{-2}^{2} x^3\,dx$$
  $$= 2\int_{0}^{2} x^4\,dx - 0 = \frac{64}{5}.$$

- $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x - 4\sin^3 x\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx - 4\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x\,dx$$
  $$= 2\int_{0}^{\frac{\pi}{2}} \cos x\,dx - 0 = 2\sin x\Big|_0^{\pi/2} = 2(1-0) = 2.$$

- $\int_{-2}^{2} ax^4 + bx^3\,dx$

Average values

Let us partition $[a, b]$ into equally space points $\{x_0, x_1, \cdots, x_n\}$ such that any two consecutive points is are $\Delta x = (b-a)/n$ apart. Let $f(x)$ be defined on $[a, b]$. As in the case of a Riemann sum, we choose in each interval $[x_{k-1}, x_k]$ an arbitrary point x_k^*. We consider the average

$$\frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{n}$$

$$= \frac{f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)}{(b-a)/\Delta x} = \frac{1}{b-a}\sum_{j=1}^{n} f(x_j^*)\,\Delta x$$

$$\to \frac{1}{b-a}\int_a^b f(t)\,dt, \qquad n \to \infty.$$

We call the

$$\overline{f} = \frac{1}{b-a}\int_a^b f(t)\,dt$$

the average value of the function f over the interval $[a, b]$.

Average values figure

The average $\overline{f}$ corresponds to deforming the area under the f into a rectangle with the same area:

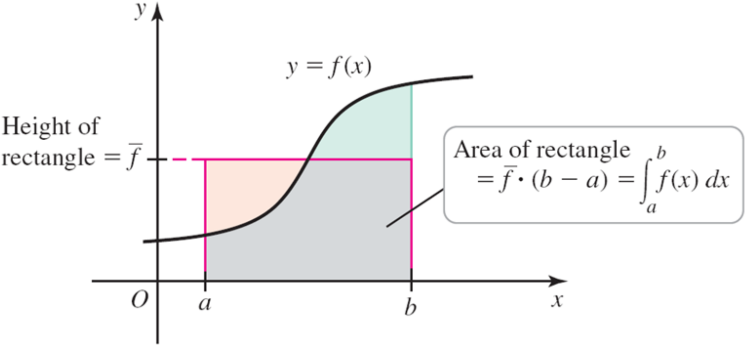


Figure: (Publisher Figure 5.51)

Average values example

A hiking trail has elevation given by

$$f(x) = 60x^3 - 650x^2 + 1200x + 4500$$

measured in feet above sea level, where x represents horizontal distance along the trail in miles, in the range $0 \leq x \leq 5$. Find the average elevation of this trail.

According to earlier discussion, the average is given by

$$\begin{aligned}\overline{f} &= \frac{1}{5}\int_0^5 60x^3 - 650x^2 + 1200x + 4500\,dx \\ &= \frac{1}{5}\left(\frac{60}{4}x^4 - \frac{650}{3}x^3 + \frac{1200}{2}x^2 + 4500\right)\Big|_0^5 = 3958 + \frac{1}{3}.\end{aligned}$$

Average values example figure

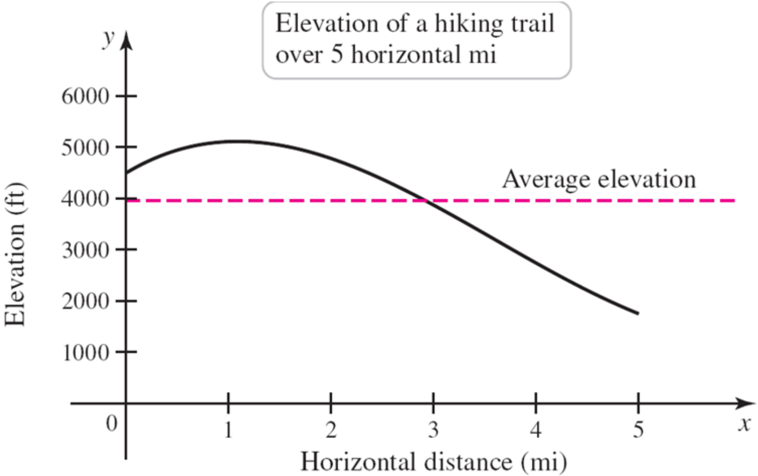


Figure: (Publisher Figure 5.52)

Mean value theorem for integrals

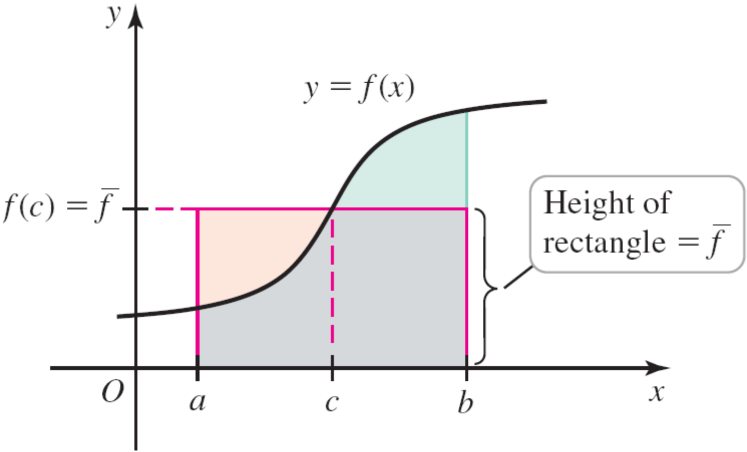


Figure: (Publisher Figure 5.53)

Basic observation

1. $\int (2x+1)(x^2+x+1)^2\,dx$

   Notice that
   $$\frac{d}{dx}(x^2+x+1)^3 = 3(x^2+x+1)(2x+1).$$
   So
   $$\int (2x+1)(x^2+x+1)^2 dx = \frac{1}{3}(x^2+x+1)^3 + C.$$

2. $\int \frac{2x+1}{\sqrt{x^2+x+1}}\,dx.$

   We have again
   $$\frac{d}{dx}\sqrt{x^2+x+1} = \frac{1}{2}(x^2+x+1)^{-1/2}(2x+1) = \frac{2x+1}{2\sqrt{x^2+x+1}}.$$
   Hence
   $$\int \frac{2x+1}{\sqrt{x^2+x+1}} dx = 2\sqrt{x^2+x+1} + C.$$

Chain Rule revisited

The above examples become much more difficult to "recognize" when compared to the previous example. We shall develop a general method by making "substitution".

Suppose $u = u(x)$, and $y = f(u(x))$. Then the chain rule implies that

$$\frac{d}{dx} f(u(x)) = \frac{df}{du}\frac{du}{dx}.$$

Thus

$$\int \frac{df}{du}\frac{du}{dx}\,dx = f(u(x)) = f(u). \tag{1}$$

On the other hand, we get directly

$$\int \frac{df}{du}\,du = f(u)$$

By comparing the above two equations, we deduce

$$\int \frac{df}{du}\frac{du}{dx}\,dx = f(u) = \int \frac{df}{du}\,du.$$

Notice the cancellation of the dx in the expression above.

Substitution examples (a)

1. $\int x\, e^{x^2}\, dx$.

   Let $u = u(x) = x^2$, then $\dfrac{du}{dx} = 2x$. It follows that

$$\int xe^{x^2}\, dx = \int \frac{1}{2} 2xe^{x^2}\, dx = \int \frac{1}{2} e^u \frac{du}{dx}\, dx$$
$$= \frac{1}{2}\int e^u\, du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C.$$

2. $\int (4x+3)^{10}\, dx$

   Put $u = 4x + 3$. Then $du = 4dx$. Hence

$$\int (4x+3)^{10}\, dx = \int u^{10} \frac{du}{4} = \frac{1}{4}\frac{1}{11}u^{11} + C$$
$$= \frac{1}{44}(4x+3)^{11} + C.$$

Substitution examples (b)

1. $\int (2x-5)(x^2-5x)^3\,dx$.

   Put $u = x^2 - 5x$. Then $du = (2x-5)dx$. Thus

$$\int (2x-5)(x^2-5x)^3 dx = \int (2x-5)u^3 \frac{du}{2x-5}$$

$$= \int u^3 du = \frac{1}{4}u^4 + C = \frac{1}{4}(x^2-5x)^4 + C.$$

2. $\int \frac{3t^2+2t}{t^3+t^2+1}\,dt$.

   Set $u = t^3 + t^2 + 1$. Thus $du = (3t^2+2t)dt$. So

$$\int \frac{3t^2+2t}{t^3+t^2+1}\,dt = \int \frac{3t^2+2t}{u}\frac{du}{3t^2+2t}$$

$$= \int \frac{du}{u} = \ln|u| + C = \ln|t^3+t^2+1| + C.$$

Substitution examples (c)

1. $\displaystyle\int x^2\sqrt{x^3+1}\,dx$.

   Set $u = x^3 + 1$. Then $du = 3x^2 dx$. And thus

$$\int x^2\sqrt{x^3+1}dx = \int x^2 u^{1/2}\frac{du}{3x^2}$$
$$= \frac{1}{3}\int u^{1/2}du = \frac{1}{3}\frac{2}{3}u^{3/2} + C = \frac{2}{9}(x^3+1)^{3/2} + C.$$

2. $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$.

   Let $u = 2x^2$. Then $du = \frac{1}{x}\,dx$. Thus we have

$$\int \frac{(\ln x)^2}{2}\,dx = \int \frac{u^2}{x}x\,du = \int u^2 du$$
$$= \frac{1}{3}u^3 + C = \frac{1}{3}(\ln|x|)^3 + C.$$

Substitution examples (d)

1. $\displaystyle\int \frac{2-x}{\sqrt{2x^2-8x+1}}\,dx.$

Set $u = 2x^2 - 8x + 1$. Then $du = (4x-8)dx$. Hence

$$\int \frac{2-x}{\sqrt{2x^2-8x+1}}\,dx = \int \frac{2-x}{\sqrt{u}}\frac{du}{4x-8}$$
$$= \frac{-1}{4}\int u^{-\frac{1}{2}}du = -\frac{1}{4}2u^{\frac{1}{2}} + C$$
$$= -\frac{1}{2}\sqrt{2x^2-8x+1} + C.$$

Solved substitution examples (a)

1.

$$\int (x^3+2)^2 3x^2\,dx \overset{u=x^3+2}{=\!=\!=} \int u^2\,du = \frac{1}{3}u^3 + c = \frac{1}{3}(x^3+2)^3 + c.$$

2.

$$\int (x^3+2)^{1/2} x^2\,dx = \frac{1}{3}\int (x^3+2)^{1/2} 3x^2\,dx = \frac{1}{3}\int u^{1/2} du$$
$$= \frac{1}{3}\frac{u^{3/2}}{3/2} + c = \frac{2}{9}(x^3+2)^{3/2} + c.$$

3.

$$\int \frac{8x^2}{(x^3+2)^3}\,dx = 8\cdot\frac{1}{3}\int (x^3+2)^{-3} 3x^2\,dx = \frac{8}{3}\int u^{-3} du$$
$$= -\frac{8}{3}\left(\frac{1}{2}u^{-2}\right) + c = \frac{-4}{3(x^3+2)^2} + c.$$

Solved substitution examples (b)

1.

$$\int \frac{x^2}{\sqrt[4]{x^3+2}}\,dx = \frac{1}{3}\int (x^3+2)^{-\frac{1}{4}} 3x^2\,dx = \frac{1}{3}\int u^{-\frac{1}{4}}\,du$$
$$= \frac{1}{3}\cdot\frac{4}{3}u^{3/4} + c = \frac{4}{9}(x^3+2)^{3/4} + c.$$

2.

$$\int 3x\sqrt{1-2x^2}\,dx \overset{u=1-2x^2}{=\!=\!=} \int 3(-\frac{1}{4})(1-2x^2)^{1/2}(-4x)\,dx$$
$$= -\frac{3}{4}\int u^{1/2}\,du + c = -\frac{1}{2}(1-2x^2)^{3/2} + c.$$

3.

$$\int x\sqrt[3]{1-x^2}\,dx = -\frac{1}{2}\int (1-x^2)^{1/3}(-2x)\,dx$$
$$= -\frac{1}{2}\cdot\frac{3}{4}(1-x^2)^{4/3} + c = -\frac{3}{8}(1-x^2)^{4/3} + c.$$

Solved substitution examples (c)

1.

$$\int \sqrt{x^2 - 2x^4}\, dx = \int x(1-2x^2)^{1/2}\, dx$$

$$= -\frac{1}{4}\int (1-2x^2)^{1/2}(-4x)\, dx = -\frac{1}{6}(1-2x^2)^{3/2} + c.$$

2.

$$\int \frac{(1+x)^2}{\sqrt{x}}\, dx = \int \frac{1+2x+x^2}{x^{1/2}}\, dx$$

$$= \int (x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} + x^{3/2})dx = 2x^{\frac{1}{2}} + \frac{4}{3}x^{3/2} + \frac{2}{5}x^{5/2} + c.$$

3.

$$\int \frac{x^2+2x}{(x+1)^2}\, dx = \int (1 - \frac{1}{(x+1)^2})\, dx = x + \frac{1}{x+1} + c'$$

$$= \frac{x^2}{x+1} + 1 + c' = \frac{x^2}{x+1} + c.$$

Solved substitution examples (d)

1.
$$\int \frac{x+3}{(x^2+6x)^{1/3}}\,dx = \frac{1}{2}\int (x^2+6x)^{-\frac{1}{3}}(2x+6)\,dx = \frac{1}{2}\int u^{-\frac{1}{3}}\,du$$
$$= \frac{1}{2}\cdot\frac{3}{2}u^{2/3}+c = \frac{3}{4}(x^2+6x)^{2/3}+c.$$

2.
$$\int \frac{dx}{2x-3} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u|+c = \frac{1}{2}\ln|2x-3|+c.$$

3.
$$\int e^x(e^x+1)^3\,dx = \int u^3\,du = \frac{u^4}{4}+c = \frac{(e^x+1)^4}{4}+c.$$

4.
$$\int \frac{dx}{e^x+1} = \int \frac{e^{-x}}{1+e^{-x}}\,dx = -\int \frac{-e^{-x}}{1+e^{-x}}\,dx = x - \ln(1+e^x)+c.$$

Definite Integrations via Substitution

Example Evaluate $\displaystyle\int_0^4 \frac{dx}{1+\sqrt{x}}$.

Put $u=\sqrt{x}$. Then $u^2=x$ and $\frac{dx}{du}=2u$. We consider the indefinite integral first. So

$$\begin{aligned}\int \frac{dx}{1+\sqrt{x}} &= \int \frac{2u\,du}{1+u} = 2\int \frac{(u+1)-1\,du}{1+u}\\ &= 2\int\left(1-\frac{1}{1+u}\right)du = 2\left(u-\ln(1+u)\right)\\ &= 2\left(\sqrt{x}-\ln\left(1+\sqrt{x}\right)\right).\end{aligned}$$

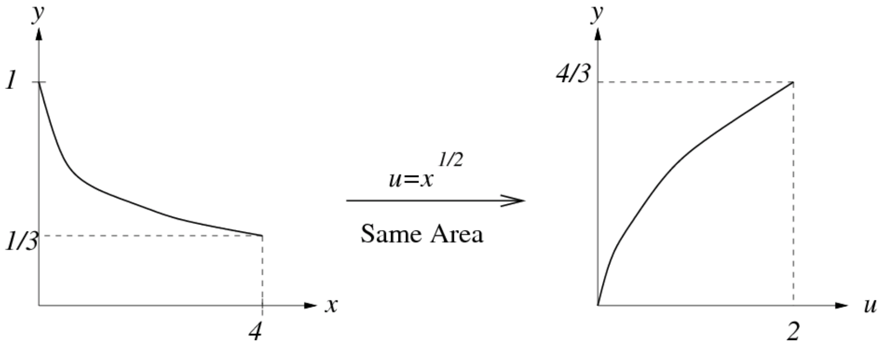
Hence

$$\begin{aligned}\int_0^4 \frac{dx}{1+\sqrt{x}} &= 2\left[\sqrt{x}-\ln\left(1+\sqrt{x}\right)\right]_0^4\\ &= 2\left(\sqrt{4}-\ln\left(1+\sqrt{4}\right)-\left(\sqrt{0}-\ln(1+\sqrt{0})\right)\right)\\ &= 2\left(2-\ln 3\right).\end{aligned}$$

Definite Integrations via Substitution (b)

On the other hand, $u = \sqrt{4} = 2$ when $x = 2$ and $u = \sqrt{0} = 0$ when $x = 0$. Hence

$$\int_0^4 \frac{dx}{1+\sqrt{x}} = \int_0^2 \frac{2u\,du}{1+u} = 2\left[u - \ln(1+u)\right]_0^2 = 2\left(2 - \ln 3\right).$$



Definite Integrations via Substitution (c)

Formula for integration by substitution with limits

$$\int_a^b g(u(x))\frac{du}{dx}\,dx = \int_{u(a)}^{u(b)} g(u)\,du.$$

THEOREM 5.7 Substitution Rule for Definite Integrals
Let $u = g(x)$, where g' is continuous on $[a, b]$, and let f be continuous on the range of g. Then

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

Definite Integrations via Substitution examples (p. 381)

1.

$$\int_0^{\pi/2} \sin^4 x \cos x \, dx = \int_0^1 u^4 \, du = \frac{u^5}{5}\Big|_0^1 = \frac{1}{5}.$$

2.

$$\begin{aligned} \int_0^4 \frac{x}{x^2+1} \, dx &= \frac{1}{2} \int_1^{17} \frac{1}{u} \, du \\ &= \frac{1}{2} \ln|u| \Big|_1^1 7 \\ &= \frac{1}{2} (\ln 17 - \ln 1). \end{aligned}$$

Definite Integrations via Substitution examples (p. 382)

1.

$$\int_0^2 2(2x+1)\,dx = \int_1^5 u\,du = \frac{u^2}{2}\Big|_1^5.$$