MATH1013 Calculus I
Assignment 1

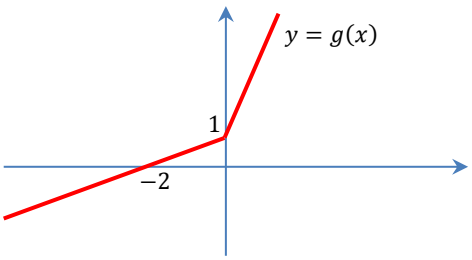
Important note:

- There are **8** problems in total. Please write your solution to each problem clearly.
- Always give **exact values** for numerical answers.
- This assignment is due on **Monday, March 11, 2013**. You are required turn in your work to the collection box labeled "MATH1013" outside the general office of the Department of MATH **(Room 3461, L25-26) no later than 17:00**. Late submission will result in score deduction.
- Peer collaboration is encouraged but **plagiarism is strictly prohibited**.

1. Let f be a function defined by the formula $f(x) = 2^{\sqrt{x+1}}$.
   (a) Find the (natural) domain of f.
   (b) Find the range of f.

(4 points)

2. The following shows the graph of a function g.



   (a) Briefly explain why g is invertible, i.e. why g^{-1} exists.
   (b) Sketch the graph of g^{-1}.
   (c) Sketch the graph of the function h, defined by $h(x) = g^{-1}(|x| + 2)$. Remember to label clearly the coordinates of the points where the graph bends.

(7 points)

3. Without using a calculator, find the value of the following:

   (a) $\arcsin\left(\cos\frac{10\pi}{3}\right)$

   (b) $\sin(2\arccos x)$

   (c) $\cos\left(\arctan\frac{x}{\sqrt{9-x^2}}\right)$

(9 points)

4. Find all real solutions to the equation $\log_{x+3}(x^2+6x+7)=1$.

(6 points)

5. Evaluate the following limits:

   (a) $\lim_{x\to+\infty}\frac{\operatorname{arccot}x}{x}$

   (b) $\lim_{x\to-\infty}\left(\sqrt{x^2+3x-10}+x\right)$

   (c) $\lim_{x\to0}x^3\sin\frac{1}{x}$

   (d) $\lim_{x\to0}\frac{\sin 4x}{\sin 2x+\sin 3x}$

   [Hint: Use the sum-to-product formula.]

(12 points)

6. Let f be the function defined by $f(x)=\begin{cases}\cos\frac{\pi x}{3} & \text{if } x<-2\\ 0 & \text{if } x=-2\\ \frac{3x^2+4x+1}{2x^2-x-10} & \text{if } x>-2 \text{ and } x\neq\frac{5}{2}\end{cases}$.

   (a) Find $\lim_{x\to a}f(x)$ for each real number a.

   (b) Find all the asymptotes for the graph of f.

(10 points)

7. In a biological experiment, the number $N(t)$ of a certain type of bacteria can be modeled by the equation $N(t)=a-be^{kt}$, where t is time measured in minutes. Initially there are 400 bacteria; after one minute, the number of bacteria grows to 700; and after one more minute, there are 850 bacteria.

   (a) Find the values of the constants a, b and k.

   (b) How will the number of bacteria behave after a very long time?

(7 points)

8. Suppose that the function $g(x)=\begin{cases}x^2+ax+b & \text{if } x<1\\ \sin\frac{\pi x}{2} & \text{if } 1\le x<2\\ \ln(x+a) & \text{if } x\ge2\end{cases}$ is continuous on $\mathbb{R}$. Find the values of the constants a and b.

(5 points)