

MATH150 Introduction to ODEs, Fall 2010-11

Hints to Week 01 Revision worksheet: Differentiations and Integrations

Name: _____ ID No.: _____ Tutorial Section: _____

Attempt at least FOUR questions from the following questions! The worksheet MUST be handed in at the end of the tutorial

(Partial solution of this worksheet will be available at the course website a week after all the tutorials)

1. **(Demonstration)** Do two/three differentiation problems for each of the categories below

- (a) Simply polynomials which may have negative powers;
- (b) simple (polynomials/rational) functions that require product/quotient rules;
- (c) (polynomials/rational) simple functions that require chain rule;
- (d) repeat the above for trigonometric functions when time allowed.

Students should be allowed time to do some class problems before turning to integration problems.

2. **(Demonstration)** Do two/three problems in integration of functions that require either one or a combination of

- (a) simple power law;
- (b) substitution;
- (c) trigonometric functions;
- (d) integration by parts.

3. **(Class work)** Find dy/dx for the following functions:

(a) $y = 5x^4 + x^2 + x$: $\underline{20x^3 + 2x + 1}$;

(b) $y = x^3 + \frac{1}{x^3}$: $\underline{3x^2 - 3x^{-4}}$;

(c) $y = (x^2 + 1)^2(2x + 3)$: $\underline{(4x(x^2 + 1)(2x + 3) + 2(x^2 + 1)^2)}$;

(d) $y = \frac{x^2 + 1}{x^2 - 1}$: $\underline{-4x(x^2 - 1)^{-2}}$;

(e) $y = \sqrt{1 + x^2}$: $\underline{\frac{x}{\sqrt{1 + x^2}}}$;

(f) $y = (1 + 3x)^4$: $\underline{12(1 + 3x)^3}$;

(g) $y = \frac{\sin x}{x}$: $\underline{\frac{x \cos x - \sin x}{x^2}}$;

(h) $y = (\ln x)^2$: $\underline{\frac{2 \ln x}{x}}$;

4. **(Class work)** Find $\int f(x)dx$ for the following functions:

(a) $\int x^3 dx$: $\underline{x^4/4 + C}$;

(b) $\int \frac{1}{\sqrt{x}} dx$: $\underline{2\sqrt{x} + C}$;

$$(c) \int (ax+b)^n dx: \quad \underline{\frac{(ax+b)^{n+1}}{a(n+1)} + C;}$$

$$(d) \int \frac{1}{(ax+b)^n} dx: \quad \underline{\frac{1}{a(1-n)}(ax+b)^{-n+1} + C \text{ for } n \neq 1, \frac{1}{a} \ln |ax+b| + C \text{ for } n = 1;}$$

$$(e) \int \frac{1}{(ax+b)} dx: \quad \underline{\frac{1}{a} \ln |ax+b| + C;}$$

$$(f) \int \sin 3x dx: \quad \underline{-\frac{1}{3} \cos 3x + C;}$$

$$(g) \int x \cos(1+x^2) dx: \quad \underline{\frac{1}{2} \sin(1+x^2) + C;}$$

$$(h) \int x \sin x dx: \quad \underline{\sin x - x \cos x + C;}$$

5. **(Supplementary problems)** Find dy/dx for the following functions:

$$(a) y = \sqrt{x}(3x-2): \quad \underline{\frac{3x-2}{2\sqrt{x}} + 3\sqrt{x};}$$

$$(b) y = (1+2x)^5: \quad \underline{10(1+2x)^4;}$$

$$(c) y = \sqrt[3]{1+x^2}: \quad \underline{\frac{2}{3}x(1+x^2)^{-\frac{2}{3}};}$$

$$(d) y = \cos 2x: \quad \underline{-2 \sin 2x;}$$

$$(e) y = \cos^2 x: \quad \underline{-\sin 2x;}$$

$$(f) y = \cos x \sin x: \quad \underline{\cos 2x;}$$

$$(g) y = \tan x: \quad \underline{\frac{1}{\cos^2 x};}$$

$$(h) y = e^{2x}: \quad \underline{2e^{2x};}$$

$$(i) y = e^x \cos x + e^{-x} \sin x: \quad \underline{(e^x + e^{-x})(\cos x - \sin x);}$$

$$(j) y = e^x \ln x: \quad \underline{e^x \left(\frac{1}{x} + \ln x \right);}$$

$$(k) y = \ln(xe^x): \quad \underline{1 + \frac{1}{x};}$$

6. **(Supplementary problems)** Find $\int f(x)dx$ for the following functions:

$$(a) \int (2\sqrt{x} + 3x^3) dx \quad \underline{\frac{4}{3}x^{\frac{3}{2}} + \frac{3}{4}x^4 + C;}$$

$$(b) \int x^2 \cos x^3 dx \quad \underline{\frac{1}{3} \sin x^3 + C;}$$

$$(c) \int \sin(1+x) dx \quad \underline{-\cos(1+x) + C;}$$

$$(d) \int x \cos(1+x^2) dx \quad \underline{\frac{1}{2} \sin(1+x^2);}$$

$$(e) \int x \ln x dx \quad \underline{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C;}$$

$$(f) \int x^2 e^x dx \quad \underline{(x^2 - 2x + x)e^x + C;}$$

$$(g) \int e^x \sin x dx \quad \underline{\frac{1}{2}e^x(\sin x - \cos x) + C}$$