MATH150 Introduction to odes, Fall 2010

Week 02 Worksheet: Complex numbers and separable odes

Name: ____________________ **ID No.:** ____________________ **Tutorial Section:**______

To receive credit, the worksheet MUST be handed in at the end of the tutorial

(*Partial solution of this worksheet will be available at the course website a week after all the tutorials*)

1. **(Demonstration)** Model some complex number problems
2. **(Demonstration)** Model one separable ode equation
3. **(Class work)** Write as a complex number $z = x + iy$ where x and y are real:

   (a) $\dfrac{1+3i}{3-2i}$: $\qquad \dfrac{-3+11i}{13}$

   (b) $\dfrac{1}{1+i} + \dfrac{1}{1-i}$: $\qquad 1$

   (c) $\dfrac{-1-2i}{-4+3i}$: $\qquad \dfrac{-2+11i}{25}$

   (d) $-(7-i)(-4-2i)(2-i)$: $\qquad 10(7-i)$

4. **(Class work)** Convert to polar form $z = r\exp(i\theta)$:

   (a) $1+\sqrt{3}i$: $\qquad z = 2\exp(i\frac{\pi}{3})$

   (b) $(\sqrt{2}+\sqrt{2}i)^7$: $\qquad z = 128\exp(i\frac{7\pi}{4})$

5. **(Class work)** Find all solutions of the following equations by writing $z = z\exp(n2\pi i)$, with n a natural number:

   (a) $z^4 = 1$ $\qquad z = \exp(i\frac{k\pi}{2})$ $\qquad k = 0, 1, 2, 3$

   (b) $z^5 = 6i$: $\qquad z = 6^{\frac{1}{5}}\exp(i\frac{(4k+1)\pi}{10})$ $\qquad k = 0, 1, 2, 3, 4, 5$

   (c) $z^5 = 1+i$: $\qquad z = 2^{\frac{1}{10}}\exp(i\frac{(8k+1)\pi}{20})$ $\qquad k = 0, 1, 2, 3, 4, 5$

6. **(Class work)** Solve for x and y where x and y are real numbers

   (a) $2y + ix = 4 + x - i$: $\qquad x = -1, y = \frac{3}{2}$

7. **(Class work)** Find the real part and the imaginary part of the following expression, where x is real:

   (a) $\exp((5+12i)x)$: $\qquad e^{5x}\cos 12x + ie^{5x}\sin 12x$

8. **(Class work)** Solve the following differential equations by separating variables

   (a) $y' = \dfrac{x^2}{y(1+x^3)}, \quad y(0) = y_0$: $\qquad \frac{y^2}{2} - \frac{1}{3}\ln(1+x^3) = \frac{y_0^2}{2}$

(b) $y' = 2x/(y + x^2 y)$, $y(0) = -2$: $\frac{y^2}{2} - \ln(1 + x^2) = 2$

(c) $y' = xy^3(1 + x^2)^{-1/2}$, $y(0) = 1$: $\frac{3-y^{-2}}{2} - (1 + x^2)^{\frac{1}{2}} = 0$

(d) $y' = 3x^2/(3y^2 - 4)$, $y(1) = 0$ (Leave as a cubic equation): $y^3 - 4y - x^3 = -1$

(e) $y' = (2 - e^x)/(3 + 2y)$, $y(0) = 0$: $y^2 + 3y - 2x + e^x = 1$