MATH150 Introduction to Ordinary Differential Equations, Spring 2010-11
Week 03 Worksheet: First Order Differential Equations (Ver. T1A)

Name: ____________________ **ID No.:** ____________________ **Tutorial Section:** _______

Attempt at least ONE question from the following questions! The worksheet MUST be handed in at the end of the tutorial

(*The question numbers refer to the main reference book by Boyce & DiPrima (B & D). Partial solution of this worksheet will be available at the course website a week after all the tutorials*)

1. **(Demonstration)** (Ex. 2.1, Q. 29 (B & D)) Consider the initial value problem

$$y' + \frac{1}{4}y = 3 + 2\cos 2t, \qquad y(0) = 0.$$

   (a) Find the solution of this IVP and describe its behaviour for large t.
   (b) Determine the value of t for which the solution first intersects the line $y = 12$.

2. **(Demonstration)** (Ex. 1.2, Q. 9 (B & D)) The falling object in Example 2 of Chapter 1.1 (B & D) satisfies the initial value problem

$$dv/dt = 9.8 - 0.2v, \quad v(0) = 0.$$

   (a) Find the time that must elapse for the object to reach 98% of its limiting velocity.
   (b) How far does the object fall in the time found in the first part? (Hint: recall that $\frac{ds}{dt} = v(t)$, where $s(t)$ is the displacement function of the object. What is $s(T) - s(0)$?)

3. **(Demonstration)** (Ex. 1.2, Q. 7 (B & D)) The field mouse population in Example 1 of Chapter 1.1 (B & D) satisfies the equation

$$dp/dt = 0.5p - 450.$$

   (a) Find the time at which the population becomes extinct if $p(0) = 850$.
   (b) Find the time of extinction if $p(0) = p_0$, where $0 < p_0 < 900$.
   (c) Find the initial population p_0 if the population is to become extinct in one year.

4. **(Demonstration)** (Ex. 2.3, Q. 7 (B & D)) Suppose that a sum S_0 is invested at an annual rate of return r compounded continuously.

   (a) Find the T required for the original sum to double in value as a function of r.
   (b) Determine T if $r = 7\%$.
   (c) Find the return rate that must be achieved if the initial investment is to double in 8 years.

5. **(Demonstration)** (Ex. 2.3, Q. 2 (B & D)) A tank initially contains 120 liters of pure water. A mixture containing a concentration of γ g/liter of salt enters the tank at a rate of 2 liters/mins, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount of salt in the tank as $t \to \infty$.

6. **(Class work)** (Ex. 2.1, Q. 31 (B & D)) Consider the IVP

$$y' - \frac{3}{2}y = 3t + 2e^t, \qquad y(0) = y_0.$$

   Find the value of y_0 that separates solutions that grow positively as $t \to \infty$ from those that grow negatively. How does the solution that corresponds to this critical value of y_0 behave as $t \to \infty$.

Answer ______________________________

7. **(Class work)** (Ex. 1.2, Q. 8 (B & D)) Consider a population p of a field mice that grows at a rate proportional to the crrent population, so that $\dfrac{dp}{dt} = rp$.

   (a) Find the rate constant r if the population doubles in 30 days.

   Answer ______________________________

   (b) Find r if the population doubles in N days.

   Answer ______________________________

8. **(Class work)** (Ex. 2.3, Q. 8 (B & D)) A young person with no initial capital invests k dollars per year at an annual rate of return r. Assume that investments are made continuously and that the return is compounded continuously.

   (a) Determine the sum $S(t)$ accumulated at any time t.

   Answer ______________________________

   (b) If $r = 7.5\%$, determine k so that §1 million will be available for retirement in 40 years.

   Answer ______________________________

   (c) If $k = \$2000$ per year, determine the return rate r that must be obtained to have \$1 million available in 40 years.

   Answer ______________________________