MATH150 Introduction to Ordinary Differential Equations, Fall 2010

Week 04 Worksheet: Applications

Name: _____ **ID No.:** _____ **Tutorial Section:**_____

Your tutor should model one or two of the following questions, and you should try to solve some of the remaing questions, time permitting. Any questions left unsolved can be worked on at home. To obtain Worksheet credit, answer sheets showing at least some solution work and your name, id number, and tutorial section (as above) MUST be handed in at the end of the tutorial. Your tutor will give you specific instructions.

(*Solution of this worksheet will be available from the tutor's website next week*)

1. A home buyer can afford to spend no more than \$800/month on mortgage payments. Suppose that the interest rate is 9% and that the term of the mortgage is 20 years. Assume that interest is compounded continuously and that payments are also made continuously.
   (a) Determine the maximum amount that this buyer can afford to borrow.
   (b) Determine the total interest paid during the term of the mortgage.

2. A sky diver weighing 82 kg (including equipment) falls vertically downward from an altitude of 1500 meters, and opens the parachute after 10 sec of free fall. Assume that the force of air resistance is $k_1|v|$, with $k_1 = 1.65$ kg/s, when the parachute is closed and $k_2|v|$, with $k_2 = 26.4$ kg/s, when the parachute is open.
   (a) Find the speed of the sky diver when the parachute opens.
   (b) Find the distance fallen before the parachute opens.
   (c) What is the limiting velocity v_L after the parachute opens?
   (d) Determine how long the sky diver is in the air after the parachute opens.
   (e) Sketch the graph of velocity versus time from the beginning of the fall until the skydiver reaches the ground.

3. Find the escape velocity for a body projected upward with an initial velocity v_0 from a point $x_0 = \xi R$ above the center of the earth, where R is the radius of the earth and ξ is a constant greater than unity. Neglect air resistance. Find the initial altitude from which the body must be launched in order to reduce the escape velocity to 85% of its value at the earth's surface.

4. Suppose that a sum S_0 is invested at an annual rate of return r compounded continuously.
   (a) Find the T required for the original sum to double in value as a function of r.
   (b) Determine T if $r = 7\%$.
   (c) Find the return rate that must be achieved if the initial investment is to double in 8 years.

5. A tank initially contains an amount S (liters) of pure water. A mixture containing a concentration γ (grams/liter) of salt enters the tank at a rate r (liters/minute), and the well-stirred mixture leaves the tank at the same rate.
   (a) Determine a differential equation for the amount of salt $M(t)$ (grams) in the tank at any time t by writing an equation for $M(t + \Delta t)$.
   (b) Solve this differential equation using an integrating factor.
   (c) Find the limiting amount of salt in the tank as $t \to \infty$, and show that this corresponds to the solution obtained by setting $dM/dt = 0$.