MATH150 Introduction to Ordinary Differential Equations, Spring 2010-11

Week 05 Worksheet: Second order equations (Ver. T1A)

Name: ____________________ **ID No.:** ____________________ **Tutorial Section:**_______

Complete at least TWO questions from the following questions! The worksheet must be handed in at the end of the tutorial. (*The question numbers refer to the main reference book by Boyce & DiPrima (B & D). Partial solution of this worksheet will be available at the course website a week after all the tutorials*)

1. **(Demonstration)** (Ex. 3.1, Q. 11 (B & D)) Solve the initial value proble and to sketch the graph and investigate its behaviour when $t \to \infty$: $6y'' - 5y' + y = 0, \quad y(0) = 4,\ y'(0) = 0,$
2. **(Demonstration)** (Ex. 3.1, Q. 21 (B & D)) Solve the initial value problem $y'' - y' - 2y = 0$, $y(0) = \alpha,\ y'(0) = 2$ and to determine the constant α so that the soluion tends to zero as $t \to \infty$.
3. **(Demonstration)** (Ex. 3.1, Q. 23 (B & D)) Determine (i) the value α below for which all solutions tend to zero as $t \to \infty$ and (ii) the value α below for which all (non-zero) solutions becomes unbounded as $t \to \infty$:
$$y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0.$$
4. **(Demonstration)** (Ex. 3.1, Q. 25 (B & D)) When time allows.
5. **(Demonstration)** (Ex. 3.3, Q. 8 (B & D)) Solve $y'' - 2y' + 6y = 0$.
6. **(Demonstration)** (Ex. 3.3, Q. 19 (B & D)) Solve $y'' - 2y' + 5y = 0, \quad y(\frac{\pi}{2}) = 0,\ y'(\frac{\pi}{2}) = 2.$
7. Solve the following initial value problems and sketch their curves:
   (a) **(Class work)** (Ex. 3.1, Q. 9 (B & D)) $y'' + y' - 2y = 0, \quad y(0) = 1, y'(0) = 1;$

   Answer ____

   (b) **(Class work)** (Ex. 3.1, Q. 14 (B & D)) $2y'' + y' - 4y = 0, \quad y(0) = 0, y'(0) = 1.$

   Answer ____

8. **(Class work)** (Ex. 3.1, Q. 22 (B & D)) Solve the initial value problem $4y'' - y = 0, \quad y(0) = 2,\ y'(0) = \beta$ and to determine the constant β so that the soluion tends to zero as $t \to \infty$.

Answer ____

9. **(Class work)** (Ex. 3.1, Q. 17 (B & D)) Find a differential equation whose general solution is given by $y = c_1 e^{2t} + c_2 e^{-3t}$, where c_1, c_2 are some constants.

Answer ____

10. **(Class work)** (Ex. 3.1, Q. 24 (B & D)) Determine (i) the value α below for which all solutions tend to zero as $t \to \infty$ and (ii) the value α below for which all (non-zero) solutions becomes unbounded as $t \to \infty$:

$$y'' + (3 - \alpha)y' - 2(\alpha - 1)y = 0.$$

Answer ____

11. **(Class work)** (Ex. 3.4, Q. 9 (B & D)) Solve $y'' + 2y' - 8y = 0$.

Answer ____

12. **(Further work)** Consider the second order linear homogeneous differential equation $6y'' + 5y' - 4y = 0$.

   (a) Find two (linearly independent) solutions of the equation by determining the two (distinct) roots of the characteristic equation.

   (b) By taking a suitable linear combination of your two solutions in part (a), find a solution $y_1(t)$ which satisfies the initial condition $y_1(0) = 1$, $y_1'(0) = 0$.

   (c) By taking a suitable linear combination of your two solutions in part (a), find a solution $y_2(t)$ which satisfies the initial condition $y_2(0) = 0$, $y_2'(0) = 1$.

   (d) Use the solutions in parts (b) and (c) and the principle of superposition to find a solution to the initial value problem

$$6y'' + 5y' - 4y = 0 \qquad y(0) = B\ ,\ y'(0) = C\ .$$

   (e) (optional) Explain why the uniqueness part of Theorem 3.2.1 (Existence and Uniqueness Theorem for standard 2nd order linear differential equations) tells us that linear combinations of $y_1(t)$ and $y_2(t)$ give all possible solutions of the equation.