MATH150 Introduction to Ordinary Differential Equations, Fall 2010

Week 06 Wksht: 2^{nd}-order linear, homogeneous and inhomogeneous ODEs (T1A)

Name: _____ **ID No.:** _____ **Tutorial Section:**_____

Your tutor should model Problems 1 & 3 and discuss how to solve Problems 5 & 6. Problem 6 can be solved either using real functions or complex exponentials. You should try to solve the non-Demonstration problems, time permitting. Any questions left unsolved can be worked on at home. To obtain Worksheet credit, answer sheets showing at least some solution work and your name, id number, and tutorial section (as above) MUST be handed in at the end of the tutorial. (*Solution of this worksheet will be available from the tutor's website next week*)

1. (**Demonstration**) (§3.4, Q. 19 B & D) Find the solution of the following initial value problem, sketch the graph of the solution, and describe its behavior for increasing t:

$$\ddot{x} - 2\dot{x} + 5x = 0, \quad x(\pi/2) = 0, \quad \dot{x}(\pi/2) = 2.$$

2. **(Class work)** (§3.4, Q. 25 B & D) Consider the initial value problem

$$\ddot{x} + 2\dot{x} + 6x = 0, \quad x(0) = 2, \quad \dot{x}(0) = \alpha \geq 0.$$

   (a) Find the solution $x = x(t)$.

   (b) Find α such that $x(1) = 0$.

Answer _____

3. (**Demonstration**) (§3.5, Q. 16 B & D) Consider the following initial value problem:

$$\ddot{x} - \dot{x} + \frac{1}{4}x = 0, \quad x(0) = 2, \quad \dot{x}(0) = b.$$

   Find the solution as a function of b and then determine the critical value of b that separates solutions that grow positively from those that eventually grow negatively.

4. **(Class work)** (§3.5, Q. 18 B & D) Consider the initial value problem

$$9\ddot{x} + 12\dot{x} + 4x = 0, \quad x(0) = a > 0, \quad \dot{x}(0) = -1.$$

(a) Solve the initial value problem.

(b) Find the critical value of a that separates solutions that become negative from those that are always positive.

Answer _____

5. **(Class work)** (§3.6, Q. 7 B & D) Find the general solution of the following differential equation:

$$2\ddot{x} + 3\dot{x} + x = t^2 + 3\sin t.$$

Answer _____

6. **(Class work)** (§3.6, Q. 18 B & D) Find the solution of the following initial value problem:

$$\ddot{x} + 2\dot{x} + 5x = 4e^{-t}\cos 2t, \quad x(0) = 1, \quad \dot{x}(0) = 0.$$

Answer _____