MATH150 Introduction to Ordinary Differential Equations, Spring 2010-11

Week 07 Worksheet: Mechanical Vibrations (Ver. T1A)

Name: _____ **ID No.:** _____ **Tutorial Section:**_____

Complete at least ONE question from the following questions! The worksheet must be handed in at the end of the tutorial

(*Solution of this worksheet will be available at the course website after all the Friday tutorials*)

1. **(Demonstration)** (§3.8, page 203, Q. 5) A mass weighing 2 lb streches a spring by 6 in. If the mass is pulled down an additional 3 in and then released, and if there is no damping, determine the position u of the mass at any time t. (Plot u against t) and find the frequence, period, and amplitude of the motion.

2. **(Demonstration)** (§3.8, page 203, Q. 10) A mass weighing 16 lb stretches a spring 3 in. The mass is attached to a viscous damper with a damping constant of 2 lb-sec/ft. If the mass is set in motion from its equilibrium position with a downward velocity of 3 in./sec, find its position u at any time t. (Plot u against t). Determine when the mass first retruns to its equilibrium position. Also find the time τ such that $|u(t)| < 0.01$ inch for all $t > \tau$.

3. **(Demonstration)** (§3.9, page 214, Q. 5/7) A mass weighing 4 lb stretches a spring 1.5 in. The mass is displaced 2 in. in the positive direction from its equilibrium position and released with no initial velocity. Assuming that there is no damping and that the mass is acted on by an external force of $2\cos 3t$ lb.

   (a) Find the position $u(t)$ of the mass from its equilibrium position. Plot a graph.

   (b) If the given external force is replaced by $4\sin \omega t$, then determine the frequency ω so that resonance occurs.

4. **(Class work)** (Ex. 3.8, Q. 1 (B & D)): Write $u = 3\cos 2t + 4\sin 2t$ in the form of $u = R\cos(\omega_0 t - \delta)$.

   Answer _____

5. **(Class work)** (§3.8, page 203, Q. 7) A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in., and then set in motion with a downward velocity of 2 ft/sec, and if there is no damping, find the position u of the mass at any time t. Determine the frequency, period, amplitude, and phase of the motion.

Answer _____

6. **(Class work)** (§3.9, page 214, Q. 9) If an undamped spring-mass system with a mass that weighs 6 lb and a spring constant 1 lb/in. is suddenly set in motion at $t = 0$ by an external force of $4\cos 7t$ lb, determine the position of the mass at any tiem and draw a graph of the displacement against t.

Answer _____

7. **(Further work)** (§3.8, page 203, Q. 11.) A spring is stretched 0.1 m by a force of 3 newtons. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 newtons when the velocity of the mass is 5 m/sec.

   (a) If the mass is pulled down 0.05 m below its equilibrium position and given an initial downward velocity of 0.1 m/sec, determine its position u at any time t.

   (b) Find the quasi frequency μ and the ratio of μ to the natural frequency of the corresponding undamped motion.

8. **(Further work)** (§3.9, page 214, Q. 6.) A mass of 5 kg stretches a hanging spring 0.1 metres. The mass is acted on by an external force of $10\sin(t/2)$ newtons, and is damped by a force which is proportional and opposite to the motion, and which is 2 newtons when the speed is .04 metres/second.

   (a) Find the spring constant k; so, force $= k \cdot$ stretch.

   (b) Find the damping constant γ; so, force $= \gamma \cdot$ speed.

   (c) Formulate the initial value problem if the mass is set in motion from its equilibrium position with a velocity of 0.03 metres/second.