MATH150 Introduction to Ordinary Differential Equations, Fall 2010

Week 10 Wksht: Series solutions **(TXXX)**

Name: ____________________ **ID No.:** ____________________ **Tutorial Section:**______

Your tutor should model Problem 1. You should try to solve Problem 2 and one of problems 3 and 4. The remaining problem can be solved at home. To obtain Worksheet credit, answer sheets showing at least some solution work and your name, id number, and tutorial section (as above) MUST be handed in at the end of the tutorial. (*Solution of this worksheet will be available from the tutor's website next week*)

For problems 1 and 2, solve the following differential equation by means of a power series about $x = 0$. Find the recurrence relation; also find the first four terms in each of two linearly independent solutions.

1. (**Demonstration**) (§5.2, page 259, problem 2) $y'' - xy' - y = 0$
2. (**Class work**) (§5.2, page 259, problem 4) $y'' + k^2x^2y = 0$; k a constant.

Answer ____

3. (§5.2, page 260, problem 21) The equation

$$y'' - 2xy' + \lambda y = 0, \quad -\infty < x < \infty,$$

where λ is a constant, is known as the Hermite equation.

(a) Find the first four terms in each of two linearly independent solutions about $x = 0$.

(b) Observe that if λ is a nonnegative even integer, then one or the other of the series solutions terminates and becomes a polynomial . Find the polynomial solutions for $\lambda = 0, 2, 4, 6$. Note that each polynomial is determined only up to a multiplicative constant.

(c) The Hermite polynomial $H_n(x)$ is defines as the polynomial solution of the Hermite equation with $\lambda = 2n$ for which the coefficient of x^n is 2^n. Find $H_0(x), H_1(x), H_2(x), H_3(x)$.

4. (§5.3, page 265, problem 10) The Chebyshev differential equation is

$$(1 - x^2)y'' - xy' + \alpha^2 y = 0, \quad \alpha \text{ a constant.}$$

(a) Determine two linearly independent solution in powers of x for $|x| < 1$.

(b) Show that if α is a nonnegative integer n, then there is a polynomial solution of degree n.

(c) Find a polynomial solution for each of the cases $\alpha = n = 0, 1, 2, 3$.

Answer ____