MATH150 Introduction to Ordinary Differential Equations, Fall 2010

Week 13 Wksht: Normal Modes / Fourier series (TXXX)

Name: _____ **ID No.:** _____ **Tutorial Section:**_____

Your tutor should model Problem 1 (Fourier series). You should try to solve Problem 2 (Fourier series) and Problem 3 (Normal model analysis). (*Solution of this worksheet will be available from the tutor's website next week*)

1. (**Demonstration**) Find the Fourier series (Fourier sine series) for the sawtooth function

$$f(x) = \frac{x}{\pi}, \quad -\pi < x < \pi.$$

   By suitably choosing x, discover the series expansion

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots.$$

2. (**Class work**) Find the Fourier series (Fourier sine series) for the square function

$$f(x) = \begin{cases} \text{-1}, & -\pi < x < 0; \\ 1, & 0 < x < \pi. \end{cases}$$

   By suitably choosing x, rediscover the series expansion

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots.$$

Answer _____

3. (**Class work**) Consider two equal masses connected by a single spring, with the masses free to move horizontally. Determine the governing equations, and the general solution in terms of normal modes. (Hint: The system will not oscillate if horizontally translated, or if moving horizontally with constant velocity.)

Answer _____