The following results may or may not be used in the midterm exam.

(a) Suppose $\binom{X_1}{X_2} \sim MN(\mu, \Sigma)$ where $\mu = \binom{\mu_1}{\mu_2}$ and $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$. Then, the conditional distribution of X_1 given $X_2 = x_2$ is

$$MN\Big(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2),\ \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\Big).$$

(b) For one-sample problem

$$n(\bar{X} - \mu)' S^{-1} (\bar{X} - \mu) \sim \frac{(n-1)p}{n-p} F_{p,n-p}.$$

(c) For two-sample problem: If $\Sigma_1 = \Sigma_2$,

$$\frac{n_1 n_2}{n_1 + n_2}(\bar{X}_1 - \bar{X}_2 - \mu_1 + \mu_2)' S_{\text{pooled}}^{-1} (\bar{X}_1 - \bar{X}_2 - \mu_1 + \mu_2) \sim \frac{(n_1 + n_2 - 2)p}{n_1 + n_2 - 1 - p} F_{p, n_1 + n_2 - p - 1},$$

where

$$S_{\text{pooled}} = \frac{1}{n_1 + n_2 - 2}\Big((n_1 - 1)S_1 + (n_2 - 1)S_2\Big).$$

In general,

$$(\bar{X}_1 - \bar{X}_2 - \mu_1 + \mu_2)' \Big(\frac{1}{n_1} S_1 + \frac{1}{n_2} S_2\Big)^{-1} (\bar{X}_1 - \bar{X}_2 - \mu_1 + \mu_2) \sim \chi_p^2$$

approximately for large n.

(d) For multiple sample problem: $B = SS_{Between}, W = SS_{Within}$ with degrees of freedom as $g - 1$ and $n - g$ respectively. The test statistic is $\Lambda^* = |W|/(|W + B|$. Under H_0,

$$-(n - 1 - \frac{p+g}{2}) \log(\Lambda^*) \sim \chi^2_{p(g-1)},$$

approximately for large n.