APPENDIX C

Example 9.1 (Example 8.1 continued)
Stock Price Data

TABLE 8.4 STOCK-PRICE DATA (WEEKLY RATE OF RETURN)

Week	Allied Chemical	Du Pont	Union Carbide	Exxon	Texaco
1	.000000	.000000	.000000	.039473	−.000000
2	.027027	−.044855	−.003030	−.014466	.043478
3	.122807	.060773	.088146	.086238	.078124
4	.057031	.029948	.066808	.013513	.019512
5	.063670	−.003793	−.039788	−.018644	−.024154
6	.003521	.050761	.082873	.074265	.049504
7	−.045614	−.033007	.002551	−.009646	−.028301
8	.058823	.041719	.081425	−.014610	.014563
9	.000000	−.019417	.002353	.001647	−.028708
10	.006944	−.025990	.007042	−.041118	−.024630
⋮	⋮	⋮	⋮	⋮	⋮
91	−.044068	.020704	−.006224	−.018518	.004694
92	.039007	.038540	.024988	−.028301	.032710
93	−.039457	−.029297	−.065844	−.015837	−.045758
94	.039568	.024145	−.006608	.028423	−.009661
95	−.031142	−.007941	.011080	.007537	.014634
96	.000000	−.020080	−.006579	.029925	−.004807
97	.021429	.049180	.006622	−.002421	.028985
98	.045454	.046375	.074561	.014563	.018779
99	.050167	.036380	.004082	−.011961	.009216
100	.019108	−.033303	.008362	.033898	.004566

...out the effects

TABLE 9.3

Variable	Maximum likelihood: Estimated factor loadings F_1	Maximum likelihood: Estimated factor loadings F_2	Maximum likelihood: Specific variances $\hat{\psi}_i = 1 - \hat{h}_i^2$	Principal components: Estimated factor loadings F_1	Principal components: Estimated factor loadings F_2	Principal components: Specific variances $\tilde{\psi}_i = 1 - \tilde{h}_i^2$
1. Allied Chemical	.684	.189	.50	.783	−.217	.34
2. Du Pont	.694	.517	.25	.773	−.458	.19
3. Union Carbide	.681	.248	.47	.794	−.234	.31
4. Exxon	.621	−.073	.61	.713	.412	.27
5. Texaco	.792	−.442	.18	.712	.524	.22
Cumulative proportion of total (standardized) sample variance explained	.485	.598		.571	.733	

The residual matrix of M.L. appcach.

$$\mathbf{R} - \hat{\mathbf{L}}\hat{\mathbf{L}}' - \hat{\Psi} = \begin{bmatrix} 0 & .005 & -.004 & -.024 & -.004 \\ .005 & 0 & -.003 & -.004 & .000 \\ -.004 & -.003 & 0 & .031 & -.004 \\ -.024 & -.004 & .031 & 0 & -.000 \\ -.004 & .000 & -.004 & -.000 & 0 \end{bmatrix}$$

The residual matrix based on P.C. factor approac

$$\mathbf{R} - \tilde{\mathbf{L}}\tilde{\mathbf{L}}' - \tilde{\Psi} = \begin{bmatrix} 0 & -.127 & -.164 & -.069 & .017 \\ -.127 & 0 & -.122 & .055 & .012 \\ -.164 & -.122 & 0 & -.019 & -.017 \\ -.069 & .055 & -.019 & 0 & -.232 \\ .017 & .012 & -.017 & -.232 & 0 \end{bmatrix}$$

Ex 9.2 EXAM DATA. n=220 p=6 m=2

$$R = \begin{array}{c} \begin{matrix} \text{Gaelic} & \text{English} & \text{History} & \text{Arithmetic} & \text{Algebra} & \text{Geometry} \end{matrix} \\ \begin{bmatrix} 1.0 & .439 & .410 & .288 & .329 & .248 \\ & 1.0 & .351 & .354 & .320 & .329 \\ & & 1.0 & .164 & .190 & .181 \\ & & & 1.0 & .595 & .470 \\ & & & & 1.0 & .464 \\ & & & & & 1.0 \end{bmatrix} \end{array}$$

TABLE 9.5 ML factor analysis.

Variable	Estimated factor loadings F_1	F_2	Communalities $\hat{h}_i^2$
1. Gaelic	.553	.429	.490
2. English	.568	.288	.406
3. History	.392	.450	.356
4. Arithmetic	.740	−.273	.623
5. Algebra	.724	−.211	.569
6. Geometry	.595	−.132	.372

TABLE 9.6 Rotated by −20°

Variable	Estimated rotated factor loadings F_1^*	F_2^*	Communalities $\hat{h}_i^{*2} = \hat{h}_i^2$
1. Gaelic	.369	.594	.490
2. English	.433	.467	.406
3. History	.211	.558	.356
4. Arithmetic	.789	.001	.623
5. Algebra	.752	.054	.568
6. Geometry	.604	.083	.372

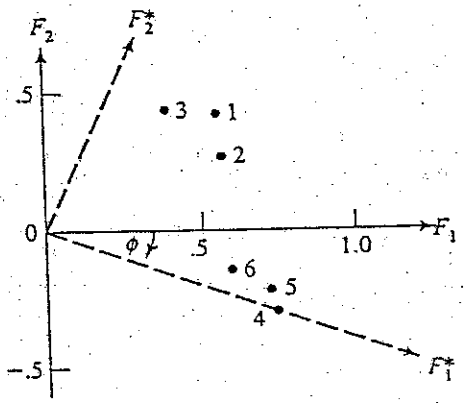


Figure 9.1 Factor rotation for test scores.

Ex 9.3 Consumer-preference data.

TABLE 9.7 P.C. factors and its rotation.

Variable	Estimated factor loadings F_1	F_2	Rotated estimated factor loadings F_1^*	F_2^*	Communalities $\hat{h}_i^2$
1. Taste	.56	.82	.02	(.99)	.98
2. Good buy for money	.78	−.52	(.94)	−.01	.88
3. Flavor	.65	.75	.13	(.98)	.98
4. Suitable for snack	.94	−.10	[.84	.43	.89
5. Provides lots of energy	.80	−.54	.97]	−.02	.93
Cumulative proportion of total (standardized) sample variance explained	.571	.932	.507	.932	

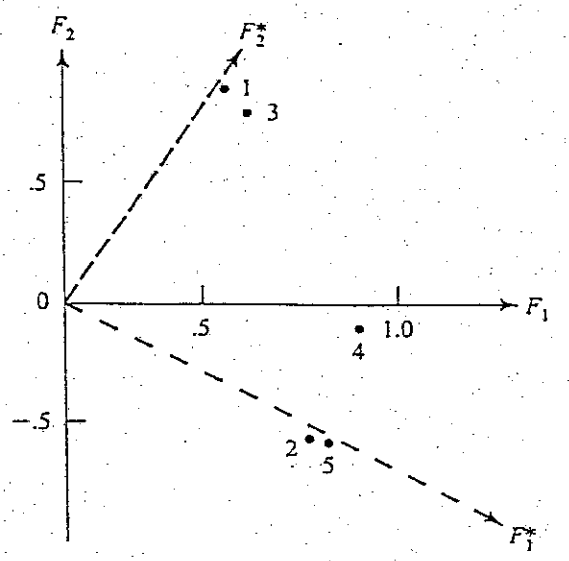


Figure 9.2 Factor rotation for hypothetical marketing data.

Ex 9.1 (Continued)

TABLE 9.8 Stock-Price Data.

Variable	Maximum likelihood estimates of factor loadings F_1	F_2	Rotated estimated factor loadings F_1^*	F_2^*	Specific variances $\hat{\psi}_i = 1 - \hat{h}_i^2$
Allied Chemical	.684	.189	.601	.377	.50
Du Pont	.694	.517	.850	.164	.25
Union Carbide	.681	.248	.643	.335	.47
Exxon	.621	−.073	.365	.507	.61
Texaco	.792	−.442	.208	.883	.18
Cumulative proportion of total sample variance explained	.485	.598	.335	.598	

Ex: Computing one factor score for ~~Exam scores~~ Stock price data (with some rotation)

$$\hat{\mathbf{L}}_z^* = \begin{bmatrix} .601 & .377 \\ .850 & .164 \\ .643 & .335 \\ .365 & .507 \\ .208 & .883 \end{bmatrix} \quad \text{and} \quad \hat{\Psi}_z = \begin{bmatrix} .50 & 0 & 0 & 0 & 0 \\ 0 & .25 & 0 & 0 & 0 \\ 0 & 0 & .47 & 0 & 0 \\ 0 & 0 & 0 & .61 & 0 \\ 0 & 0 & 0 & 0 & .18 \end{bmatrix}$$

The vector of standardized observations,

$$\mathbf{z}' = [.50, -1.40, -.20, -.70, 1.40]$$

yields the following scores on factors 1 and 2:

Weighted least squares (9-50):

$$\hat{\mathbf{f}} = (\hat{\mathbf{L}}_z^{*\prime}\hat{\Psi}_z^{-1}\hat{\mathbf{L}}_z^*)^{-1}\hat{\mathbf{L}}_z^{*\prime}\hat{\Psi}_z^{-1}\mathbf{z} = \begin{bmatrix} -1.8 \\ 2.0 \end{bmatrix}$$

Regression (9-58):

$$\hat{\mathbf{f}} = \hat{\mathbf{L}}_z^{*\prime}\mathbf{R}^{-1}\mathbf{z} = \begin{bmatrix} .187 & .657 & .222 & .050 & -.210 \\ .037 & -.185 & .013 & .107 & .864 \end{bmatrix} \begin{bmatrix} .50 \\ -1.40 \\ -.20 \\ -.70 \\ 1.40 \end{bmatrix} = \begin{bmatrix} -1.2 \\ 1.4 \end{bmatrix}$$

EX 9.4
Chicken-bone Data.

$n = 276.$ $p = 6.$ $m = 3$ (?).

Head: $\begin{cases} X_1 = \text{skull length} \\ X_2 = \text{skull breadth} \end{cases}$

Leg: $\begin{cases} X_3 = \text{femur length} \\ X_4 = \text{tibia length} \end{cases}$

Wing: $\begin{cases} X_5 = \text{humerus length} \\ X_6 = \text{ulna length} \end{cases}$

The sample correlation matrix

$$R = \begin{bmatrix} 1.000 & .505 & .569 & .602 & .621 & .603 \\ .505 & 1.000 & .422 & .467 & .482 & .450 \\ .569 & .422 & 1.000 & .926 & .877 & .878 \\ .602 & .467 & .926 & 1.000 & .874 & .894 \\ .621 & .482 & .877 & .874 & 1.000 & .937 \\ .603 & .450 & .878 & .894 & .937 & 1.000 \end{bmatrix}$$

TABLE 9.10 FACTOR ANALYSIS OF CHICKEN-BONE DATA

Principal Component

Variable	Estimated factor loadings			Rotated estimated loadings			
	F_1	F_2	F_3	F_1^*	F_2^*	F_3^*	$\hat{\psi}_i$
1. Skull length	.741	.350	.573	.355	.244	.902	.00
2. Skull breadth	.604	.720	−.340	.235	.949	.211	.00
3. Femur length	.929	−.233	−.075	.921	.164	.218	.08
4. Tibia length	.943	−.175	−.067	.904	.212	.252	.08
5. Humerus length	.948	−.143	−.045	.888	.228	.283	.08
6. Ulna length	.945	−.189	−.047	.908	.192	.264	.07
Cumulative proportion of total (standardized) sample variance explained	.743	.873	.950	.576	.763	.950	

Maximum Likelihood

Variable	Estimated factor-loadings			Rotated estimated loadings			
	F_1	F_2	F_3	F_1^*	F_2^*	F_2^*	$\hat{\psi}_i$
1. Skull length	.602	.214	.286	.467	.506	.128	.51
2. Skull breadth	.467	.177	.652	.211	.792	.050	.33
3. Femur length	.926	.145	−.057	.890	.289	.084	.12
4. Tibia length	1.000	.000	−.000	.936	.345	−.073	.00
5. Humerus length	.874	.463	−.012	.831	.362	.396	.02
6. Ulna length	.894	.336	−.039	.857	.325	.272	.09
Cumulative proportion of total (standardized) sample variance explained	.667	.738	.823	.559	.779	.823	

Checking large/small of residual matrix.

$$\mathbf{R} - \hat{\mathbf{L}}_z\hat{\mathbf{L}}_z' - \hat{\Psi}_z = \begin{bmatrix} .000 & & & & & \\ -.000 & .000 & & & & \\ -.003 & .001 & .000 & & & \\ .000 & .000 & .000 & .000 & & \\ -.001 & .000 & .000 & .000 & .000 & \\ .004 & -.001 & -.001 & .000 & -.000 & .000 \end{bmatrix}$$

Checking Outliers.

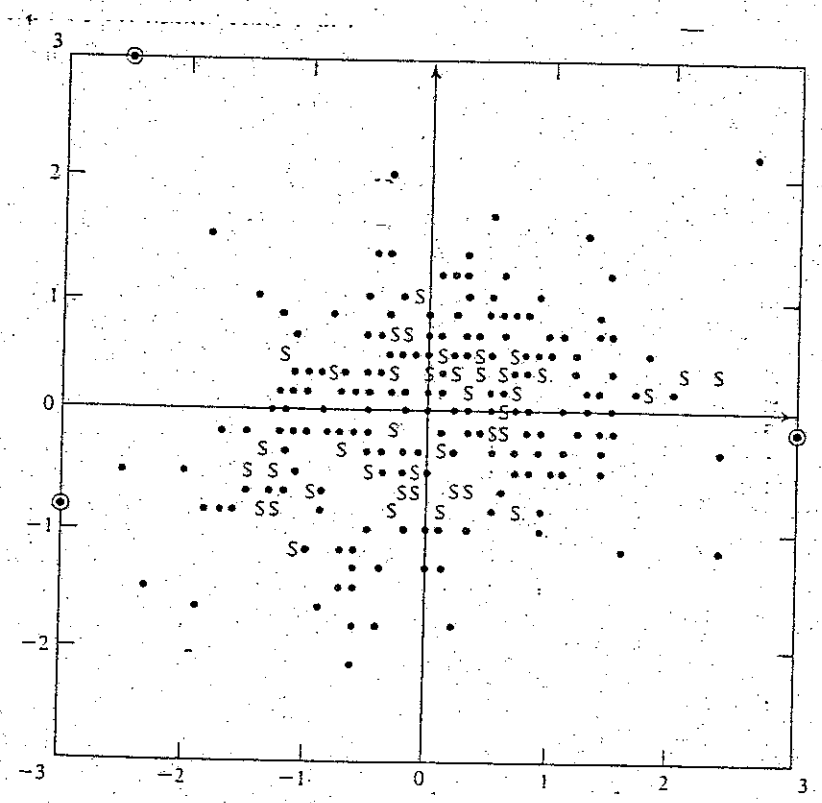


Figure 9.5 Factor scores for the first two factors of chicken-bone data.

Compairing the two approaches (P.C. & M.L.)

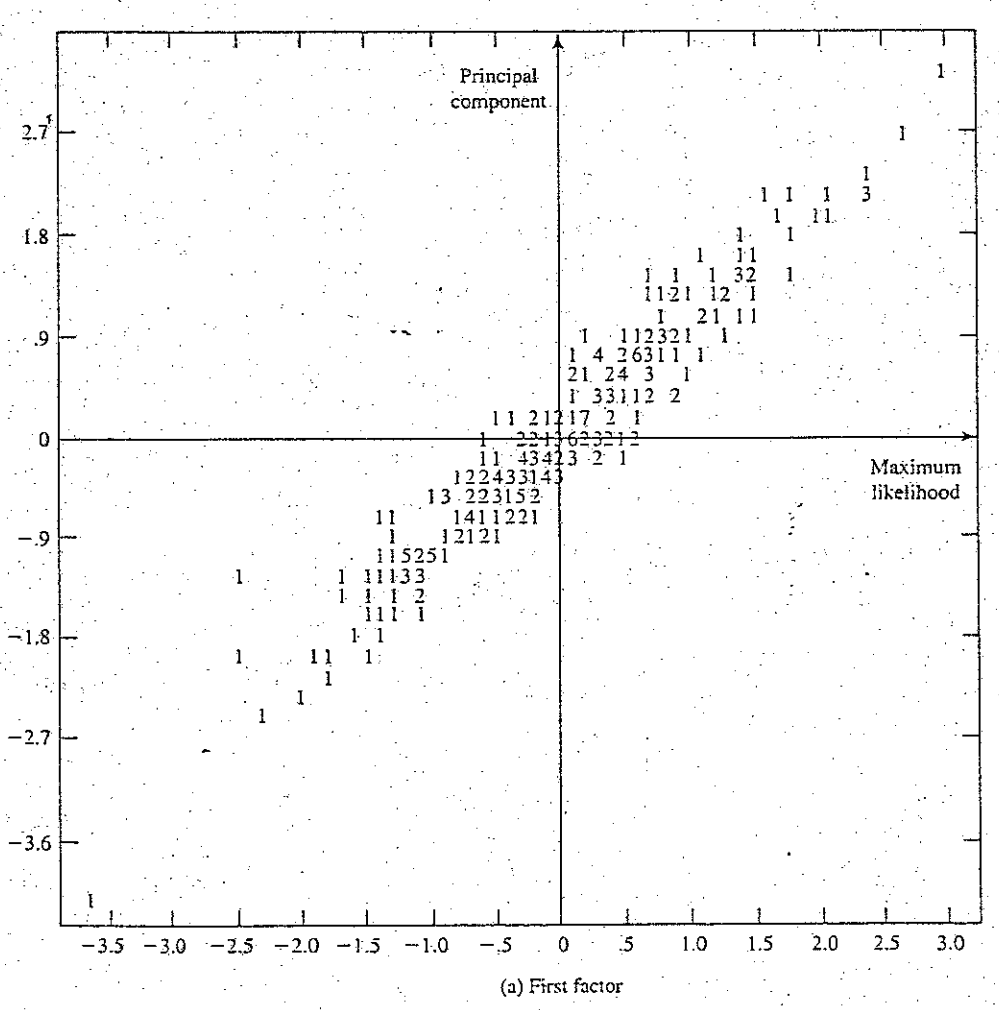
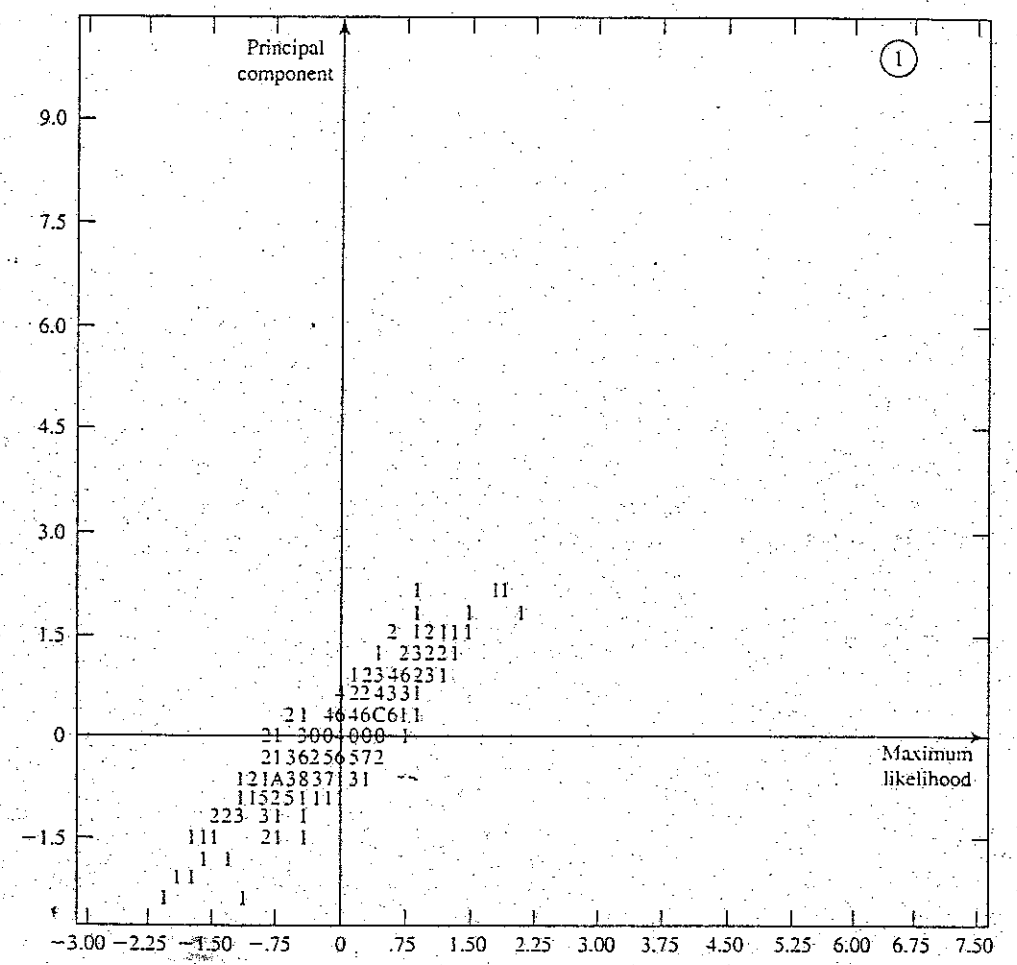
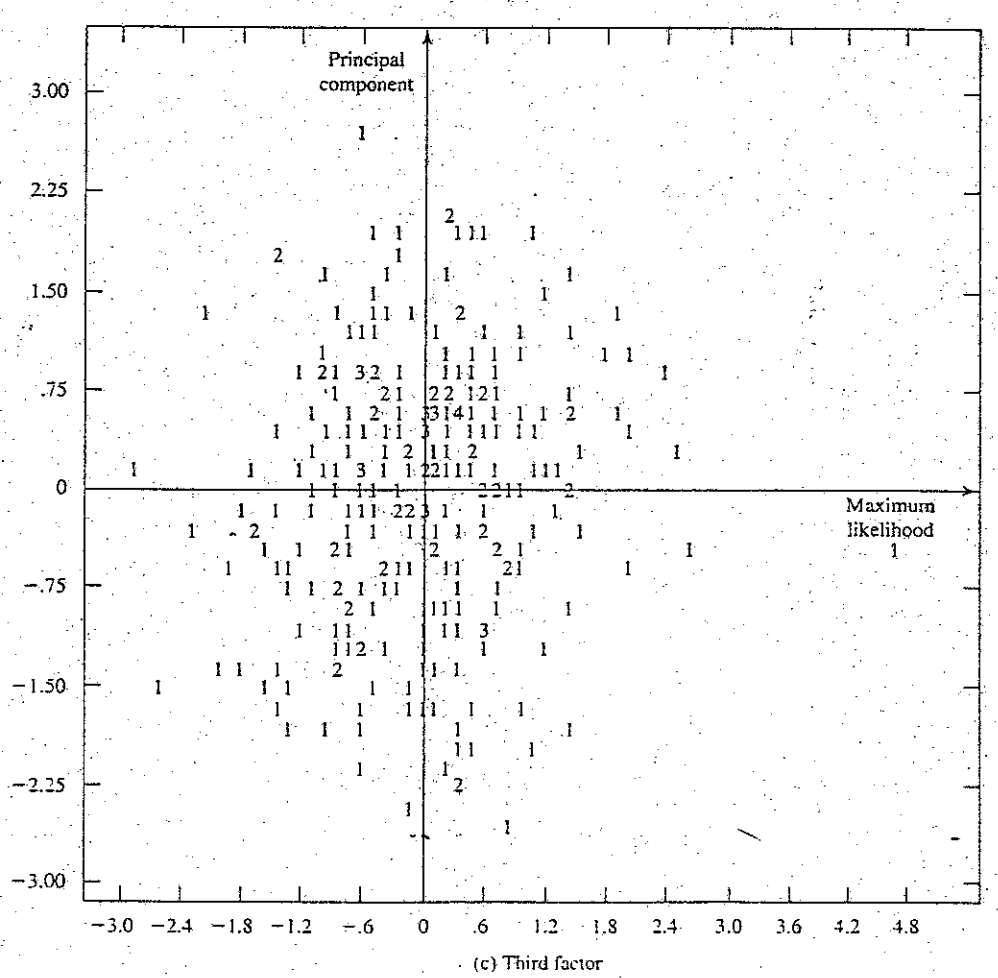


Figure 9.6 Pairs of factor scores for the chicken-bone data. (Loadings are estimated by principal component and maximum likelihood methods.)



(b) Second factor



(c) Third factor

Figure 9.6 *(continued)*

Split the data into half.
$n_1 = 137$ $n_2 = 139$

$$\mathbf{R}_1 = \begin{bmatrix} 1.000 & & & & & \\ .696 & 1.000 & & & & \\ .588 & .540 & 1.000 & & & \\ .639 & .575 & .901 & 1.000 & & \\ .694 & .606 & .844 & .835 & 1.000 & \\ .660 & .584 & .866 & .863 & .931 & 1.000 \end{bmatrix}$$

and

$$\mathbf{R}_2 = \begin{bmatrix} 1.000 & & & & & \\ .366 & 1.000 & & & & \\ .572 & .352 & 1.000 & & & \\ .587 & .406 & .950 & 1.000 & & \\ .587 & .420 & .909 & .911 & 1.000 & \\ .598 & .386 & .894 & .927 & .940 & 1.000 \end{bmatrix}$$

TABLE 9.11

	First set ($n_1 = 137$ observations) Rotated estimated factor loadings				Second set ($n_2 = 139$ observations) Rotated estimated factor loadings			
Variable	F_1^*	F_2^*	F_3^*	$\tilde{\psi}_i$	F_1^*	F_2^*	F_3^*	$\tilde{\psi}_i$
1. Skull length	.360	.361	(.853)	.01	.352	(.921)	.167	.00
2. Skull breadth	.303	(.899)	.312	.00	.203	.145	(.968)	.00
3. Femur length	(.914)	.238	.175	.08	(.930)	.239	.130	.06
4. Tibia length	.877	.270	.242	.10	.925	.248	.187	.05
5. Humerus length	.830	.247	.395	.11	.912	.252	.208	.06
6. Ulna length	(.871)	.231	.332	.08	(.914)	.272	.168	.06
Cumulative proportion of total (standardized) sample variance explained	.546	.743	.940		.593	.780	.962	