# MATH4424 Homework1

**Question 1**. Consider a bivariate normal population with $\mu_1 = 0, \mu_2 = 2, \sigma_{11} = 2, \sigma_{22} = 1$ and $\rho_{12} = 0.5$.

(a). Write out the bivariate normal density.

(b). Write out the squared generalized distance expression $(\mathbf{x} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu})$ as a function of $x_1$ and $x_2$.

(c). Determine (and sketch) the constant-density contour that contains 50% of the probability. ■

**Question 2**. Let $\mathbf{X}$ be $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\mu}' = [-3, 1, 4]$ and

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Which of the following variables are independent? Explain.

(a). $X_1$ and $X_2$.

(b). $X_2$ and $X_3$.

(c). $(X_1, X_2)$ and $X_3$.

(d). $(X_1 + X_2)/2$ and $X_3$.

(e). $X_2$ and $X_2 - \frac{5}{2}X_1 - X_3$. ■

**Question 3**. Let $\mathbf{X}$ be $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\mu}' = [2, -3, 1]$ and

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

(a). Find the distribution of $3X_1 - 2X_2 + X_3$.

(b). Relabel the variables if necessary, and find a $2 \times 1$ vector $\mathbf{a}$ such that $X_2$ and $X_2 - (X_1, X_3)\mathbf{a}$ are independent. ■

**Question 4**. Specify each of the following.

(a). The conditional distribution of $X_1$, given that $X_2 = x_2$ for the joint distribution in **Question 1**.

(b). The conditional distribution of $X_2$, given that $X_1 = x_1$ and $X_3 = x_3$ for the joint distribution in **Question 2**.

(c). The conditional distribution of $X_3$, given that $X_1 = x_1$ and $X_2 = x_2$ for the joint distribution in **Question 3**. ■