

$$(\mathbb{R}, \mathcal{F}, P) = (\mathbb{R}^N, \mathcal{B}^N, P).$$

From Kolmogorov's extension procedure,

$$\mathcal{B}^N = \sigma(\{A_n : n = 1, 2, \dots\})$$

$$A_n = B_1 \times B_2 \times \dots \times B_n \times \mathbb{R} \times \mathbb{R} \times \dots, \quad B_i \in \mathcal{B}.$$

Thus, for $\forall A \in \mathcal{B}^N$, $\exists A_n \subset A$.

$$A_1 \supset A_2 \supset A_3 \dots \rightarrow A_n.$$

$$TA = A \Leftrightarrow T^n A = A.$$

$$\begin{aligned} T^n A &= \{(w_n, w_{n+1}, \dots) : w \in A\} \\ &= \{(x_n, x_{n+1}, \dots) : w \in A\} \end{aligned}$$

$$A = \{(w_0, w_1, \dots) : w \in A\} = \{(x_0, x_1, \dots) : w \in A\}$$

$$A_n = \{(w_0, w_1, \dots) : w \in B_0 \times B_2 \times \dots \times B_n \times \mathbb{R} \times \mathbb{R} \times \dots\}$$

$$= \{(x_0, x_1, \dots) : w \in B_0 \times B_2 \times \dots \times B_n \times \mathbb{R} \times \mathbb{R} \times \dots\}$$

$$= \{(w_0, w_1, \dots, w_n) \in B_0 \times B_2 \times \dots \times B_n\}$$

$$= \{(x_0, x_1, \dots, x_n) \stackrel{(w)}{\in} B_0 \times B_2 \times \dots \times B_n\}$$

$$P(A) = P(A \cap T^{n+1} A) = \lim_{n \rightarrow \infty} P(A_n \cap T^{n+1} A_n)$$

$$= \lim_{n \rightarrow \infty} P(\{(x_0, x_1, \dots, x_n) : w \in B_0 \times \dots \times B_n\} \cap \{(x_{n+1}, x_{n+2}, \dots) : w \in A\})$$

$$= \lim_{n \rightarrow \infty} P(\{(x_0, x_1, \dots, x_n) : w \in B_0 \times \dots \times B_n\} \times P(\{(x_{n+1}, x_{n+2}, \dots) : w \in A\}))$$

$$= \lim_{n \rightarrow \infty} P(A_n) P(A) = P(A) \Rightarrow P(A) = 0 \text{ or } 1.$$