

① if $\{\eta_t\}$ is i.i.d.

$$Y_t = g(\eta_t, \eta_{t-1}, \dots)$$

is stationary and ergodic.

② Let T be a shift operator (or a measure preserving map). s.t.

$$X_n = T^n X_0 \quad [X_n(\omega) = X_0(T^n(\omega))]$$

$$\begin{aligned} Y_t &= g(X_t, X_{t-1}, \dots) \\ &= g(T^t X_0, T^{t-1} X_0, \dots) \end{aligned}$$

$$\begin{aligned} Y_t(\omega) &= g(X_0(T^t(\omega)), X_1(T^t(\omega)), \dots) \\ &= g(X_0, X_1, \dots)(T^t(\omega)) \\ &= Y_0(T^t(\omega)) \end{aligned}$$

So $Y_t = T^t Y_0$ is stationary and ergodic.

Since T is measure preserving map and ergodic.