

$$\mathcal{F}_n' = \sigma(X_n, X_{n+1}, \dots)$$

$$\mathcal{T} = \bigcap_n \mathcal{F}_n' \text{ --- remote future tail } \sigma\text{-field.}$$

Kolmogorov's 0-1 Law:

If $\{X_n\}$ are independent, and $A \in \mathcal{T}$,
then $P(A) = 0$ or 1 .

$$T^{-1}A = A$$

$$TA = T(T^{-1}A) = A$$

$$\{TW : W \in A\} = \{W : W \in A\}$$

$$\{W : W \in A\} = \{(w_1, w_2, \dots) : W \in A\} \in \mathcal{F}_1'$$

$$T^2A = TA = A$$

$$\{W : W \in A\} = \{(w_2, w_3, \dots) : W \in A\} \in \mathcal{F}_2'$$

⋮

$$\{W : W \in A\} = \{(w_n, w_{n+1}, \dots) : W \in A\} \in \mathcal{F}_n'$$

$$\Rightarrow A \in \bigcap_{n=1}^{\infty} \mathcal{F}_n'$$