5.2. Local QMLE for ARMA-GARCH/IGARCH

Looking at the main issue again:

$$E \sup_{\Theta} A_t(\theta) < \infty,$$

where

$$A_t(\theta) = \frac{\varepsilon_t^2(\gamma)}{h_t(\theta)} \left[\frac{1}{h_t(\theta)} \frac{\partial h_t(\theta)}{\partial \theta}\right] \left[\frac{1}{h_t(\theta)} \frac{\partial h_t(\theta)}{\partial \theta'}\right].$$

Note that $\varepsilon_t^2(\gamma_0)/h_t(\theta_0) = \eta_t^2$ is independent of $\mathcal{F}_{t-1}$.

If we restrict the estimator on the subspace $\Theta_n = \{\theta : \|\theta - \theta_0\| \leq M/\sqrt{n}\}$ for any fixed $M > 0$,

$\varepsilon_t^2(\gamma)/h_t(\theta) \sim \eta_t^2$.

Thus, $E\varepsilon_t^2 < \infty$ may be sufficient.

Since the tail index of the IGARCH is 2, $h_t(\theta)$ may be able to reduce a little bit of the required moment of ε_t in $\left[\frac{1}{h_t(\theta)}\frac{\partial h_t(\theta)}{\partial\theta}\right]$.

Thus, the asymptotic normality of the local QMLE holds for the ARMA-IGARCH model.

Using $\widehat{\theta}_{sn}$ as an initial estimator of θ_0,

we obtain the local MLE through the following one-step iteration:

$$\widehat{\theta}_n = \widehat{\theta}_{sn} - \Big[\sum_{t=1}^{n}\frac{\partial^2\tilde{l}_t(\widehat{\theta}_{sn})}{\partial\theta\partial\theta'}\Big]^{-1}\sum_{t=1}^{n}\frac{\partial\tilde{l}_t(\widehat{\theta}_{sn})}{\partial\theta}.$$

Theorem 4.1 *Suppose that Assumptions 2.1-2.3 and 3.1-3.2 hold and that (2.6) or the condition of Theorem 2.1 (iii) is satisfied. If $E\eta_t^4 < \infty$ and $J > 0$, then*

$$\sqrt{n}(\widehat{\theta}_n - \theta_0) \longrightarrow_{\mathcal{L}} N(0, \Sigma^{-1}\Omega\Sigma^{-1}),$$

where $\Sigma = E[U_t(\theta_0)U_t'(\theta_0)]$ *and* $\Omega = E[U_t(\theta_0)JU_t'(\theta_0)]$.

Efficiency

5. Concluding Remarks

Given a data set, different estimators may give different results in practice.

To see if the QMLE should be used, it will be helpful to estimate the tail index of the data.

If it is greater than 4, then the global QMLE can be used.

If it is in $[2, 4]$, a two-step estimator should be considered, i.e., first obtaining a self-weighted QMLE and then using it to obtain the QMLE via a one-step iteration.

The self-weighted principle:

M- and quantile-estimation,

threshold AR-ARCH models, DTAR models and multivariate time series models.