MATH 1003 Calculus and Linear Algebra (Lecture 1)

Maosheng Xiong
Department of Mathematics, HKUST

Simple Interest

Definition

Let r be the (annual) interest rate. Suppose we invest/borrow a sum of money P through a certain financial instrument, then after t years, the amount you will receive/pay, A, is given by

$$A = P + I = P + Prt = P(1 + rt)$$

where $I = Prt$ is the amount of interest received/owed.

Simple Interest

Definition

$$A = P(1 + rt), \quad I = Prt$$

- I=interest
- A=amount, or future value
- P=principal, or present value
- r=annual simple interest rate
- t=time in years

Remark

Interest rate is the percentage gain/loss of money over a year ($t = 1$) due to a financial instrument:

$$r = \frac{I}{P} \times 100\%,$$

where the principal P, is often referred to as the present value and A as the future value. If the interest rate r is set to be 5%, then

- the future value of \$100 given to you now is worth $100(1 + 0.05) = \$105$ after one year;
- the present value of \$100 given to you after one year is worth $100 \div (1 + 0.05) = \$95.24$ now.

Example 1

Example

Find the total amount to be received for a deposit of RMB10000 after three months and a year based on the following table from BEA

Period	3 months	6 months	1 year
Interest rate	2.4 p.a.	2.5 p.a.	2.7 p.a.

Remark

p.a. stands for per annum

Example 1

Solution

- 3-month:
$$A = P(1+rt) = 10000\left(1+\frac{2.4}{100}\times\frac{3}{12}\right) = (\text{RMB})10060$$
- 1-year:
$$A = P(1+rt) = 10000\left(1+\frac{2.7}{100}\times 1\right) = (\text{RMB})10270$$

Example 2

Example

If you want to earn an annual rate of 20% on your investments, how much should you pay for a note that will be worth \$5000 in 9 months?

Example 2

Solution

$$5000 = P(1+\frac{20}{100}\times\frac{9}{12})$$
$$\Rightarrow P = \$4347.8$$

Example 3 Treasure Bills

Example

T-bills (Treasury bill) are one of the instruments the U.S. Treasury Department uses to finance the public debt. If you buy a 180-day T-bill with a maturity value of \$10000 for \$9800, what annual simple interest rate you will earn?

Remark

For the convenience of calculation, here one year is "defined" to be 360 days.

Example 3 Treasure Bills

Solution

$$10000 = 9800\left(1 + r \cdot \frac{180}{360}\right)$$

$$\Rightarrow r = 0.0408 = 4.08\%$$

Example 4 Interest Rate Earned on an Investment

Example

You sell an old car to our friend and accept a 270-day note for \$3500 at 10% simple interest rate as payment. (Both principal and interest will be paid at the end of 270 days.) Sixty days later you find that you need the money and sell the note to a third party for \$3550. What annual interest rate will the third party receive for the investment?

Example 4 Interest Rate Earned on an Investment

Solution

The future value of the 270-day note

$$3500\left(1 + 0.1 \times \frac{270}{360}\right) = \$3762.5$$

The third party paid \$3550 and then after $270 - 60 = 210$ days, he/she will obtain \$3762.5.

Let r be the annual interest rate for this investment. Then we have

$$3762.5 = 3550\left(1 + r \cdot \frac{210}{360}\right)$$

$$\Rightarrow r = 0.1026 = 10.26\%$$

Example 5 Stock Investments (Optional)

Example

The brokerage firm charge commissions based on the amount of the trade. The following table shows the commission schedule for one of these firms.

Transaction Size	Commission
\$0-\$2499	\$29+1.6% of principal
\$2500-\$9999	\$49+0.8% of principal
\$10000+	\$99+0.3% of principal

An investor purchases 50 shares of a stock at \$47.52 per share. After 200 days, the investor sells the stock for \$52.19 per share. Using the above commission schedule, find the annual rate of interest earned by this investment.

Example 5 Stock Investments (Optional)

The values of 50 shares of stocks: $50 \times 47.52 = \$2376$
The commission for buying those 50 shares of stocks:

$$29 + 1.6\% \times 2376 = \$67.02$$

Therefore, the total amount of investment is
$2376 + 67.02 = \$2443.02$.

Example 5 Stock Investments (Optional)

The values of 50 shares of stocks after 200 days:
$50 \times 52.19 = \$2609.5$
The commission for selling those 50 shares of stocks:

$$49 + 0.8\% \times 2609.5 = \$69.88$$

Therefore, the total amount of money obtained by selling stocks is
$2609.5 - 69.88 = \$2539.62$

Example 5 Stock Investments (Optional)

Let r be the annual interest rate earned by this investment. Then we have

$$2539.62 = 2443.02\left(1 + r \cdot \frac{200}{360}\right)$$

Solving, we get $r = 0.0712 = 7.12\%$.