# MATH 1003 Calculus and Linear Algebra (Lecture 14)

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## Finding Derivatives using Differentiation Rules

Given a function $f(x)$, the most primitive way to compute its derivative is to evaluate the limit of its difference quotient, which may be quite difficult if $f(x)$ is a complicated expression in $x$. Therefore, we will develop some differentiation rules to facilitate the computation of derivatives. They are

- Power rule
- Product rule
- Quotient rule
- Chain rule



## Power Rule

In this lecture, we will learn the first and the most basic differentiation rule - Power rule:

**Theorem**

*(Power Rule) If $y = f(x) = x^n$ where $n$ is a real number, then*

$$f'(x) = nx^{n-1}.$$



## Power Rule - Exercises

**Example**

Find $f'(x)$ for each of the following functions:

(a) $f(x) = 1$ (More generally, $f(x) = k$, where $k$ is a constant.)

(b) $f(x) = x^5$

(c) $f(x) = x^{3/2}$

(d) $f(x) = x^{-3}$

(e) $f(x) = \dfrac{1}{\sqrt[3]{x}}$

(f) $f(x) = x^{\sqrt{2}}$.

## Basic Differentiation Properties

Besides the differentiation rules, we also need to learn two basic differentiation properties that are extremely useful in the computation of derivatives:

**Theorem**

*If $y = f(x) = ku(x)$, where $k$ is a constant, then $f'(x) = ku'(x)$.*
*If $y = f(x) = u(x) \pm v(x)$, then $f'(x) = u'(x) \pm v'(x)$.*

**Examples**

- Suppose $f(x) = 3x^5$. Then $f'(x) = 3(x^5)' = 3(5x^4) = 15x^4$.
- Suppose $f(x) = 2x^4 + 2x^3 - 3x$. Then

$$f'(x) = 2(x^4)' + 2(x^3)' - 3(x)'$$

$$\Rightarrow f'(x) = 8x^3 + 6x^2 - 3.$$



## Exercises

**Example**

Find the derivative for each of the following functions:

(a) $f(x) = 3x^4 - 2x^3 + x^2 - 5x + 7$

(b) $g(t) = 3 - \frac{5}{t^2}$

(c) $u = 6v^4 - \sqrt[5]{v}$

(d) $y = \frac{3}{5x^4} + \frac{1}{\sqrt{x}} - \frac{x^2}{2}$

(e) $h(s) = \frac{s^2 + 25}{s^2}$



## An Application of Derivatives in Physics

**Example**

An object moves along the $y$ axis so that its position at time $x$ is

$$f(x) = x^3 - 6x^2 + 9x$$

(a) Find the velocity function $v$.

(b) Find the velocity at $x = 2$ and $x = 5$.

(c) Find the time(s) when the velocity is 0.



## An Application of Derivatives in Physics

**Solution**

(a) $v(x) = f'(x) = 3x^2 - 12x + 9$

(b) $v(2) = -3$ and $v(5) = 24$.

(c) $v(x) = 0$ implies $3x^2 - 12x + 9 = 0$. Hence

$$3(x-1)(x-3) = 0 \Rightarrow x = 1, 3$$

### Example

Suppose $f(x) = 2x^3 - 9x^2 + 12x - 54$.

(a) Find $f'(x)$.

(b) Find the equation of the tangent line of $y = f(x)$ at $x = 3$.

(c) Find the value(s) of $x$ such that the tangent line at $x$ is horizontal.



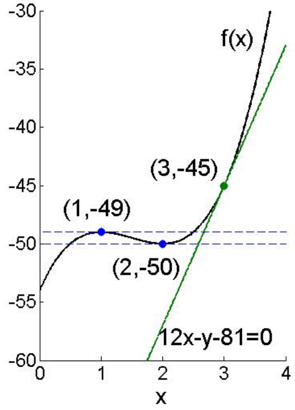


### Solution

(a) $f'(x) = 6x^2 - 18x + 12$.

(b) The slope of the tangent line at $x = 3$ is $f'(3) = 12$. Moreover, the tangent line passes through $(3, f(3)) = (3, -45)$. Then the equation of the tangent line is

$$\frac{y - (-45)}{x - 3} = 12 \Rightarrow 12x - y - 81 = 0$$




(c) $f'(x) = 0$ implies $6x^2 - 18x + 12 = 0$. Hence

$$6(x-1)(x-2) = 0 \Rightarrow x = 1, 2$$