MATH 1003 Calculus and Linear Algebra (Lecture 26)

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Examples

Example

Find each indefinite integral:

(a) $\int 2x^2 e^{x^3}\, dx$

(b) $\int \frac{x}{1+x^2}\, dx$

(c) $\int x^3\sqrt{1+x^4}\, dx$

(d) $\int \frac{1}{x(\ln x)^2}\, dx$

(e) $\int \frac{x}{\sqrt{x+1}}\, dx$

Examples

Example

Find each indefinite integral:

(a) $\int 2x^2 e^{x^3}\, dx \ (u = x^3)$

(b) $\int \frac{x}{1+x^2}\, dx \ (u = 1+x^2)$

(c) $\int x^3\sqrt{1+x^4}\, dx \ (u = 1+x^4)$

(d) $\int \frac{1}{x(\ln x)^2}\, dx \ (u = \frac{1}{x})$

(e) $\int \frac{x}{\sqrt{x+1}}\, dx \ (u = x+1)$

Examples

Solutions for (a) and (b)

(a) Let $u = x^3$. Then $du = 3x^2\, dx$. Hence $x^2\, dx = \frac{1}{3}\, du$. Substituting, we get

$$\int 2x^2 e^{x^3}\, dx = \int \frac{2}{3} e^u\, du = \frac{2}{3} e^u + C$$

$$\Rightarrow \int 2x^2 e^{x^3}\, dx = \frac{2}{3} e^{x^3} + C$$

(b) Let $u = 1+x^2$. Then $du = 2x\, dx$. Hence $x\, dx = \frac{1}{2}\, du$. Substituting, we get

$$\int \frac{x}{1+x^2}\, dx = \int \frac{1}{2u}\, du = \frac{1}{2}\ln|u| + C$$

$$\Rightarrow \int \frac{x}{1+x^2}\, dx = \frac{1}{2}\ln|1+x^2| + C$$

Examples

Solutions for (c) and (d)

(c) Let $u = 1 + x^4$. Then $\mathrm{d}u = 4x^3\,\mathrm{d}x$. Hence $x^3\,\mathrm{d}x = \frac{1}{4}\,\mathrm{d}u$. Substituting, we get

$$\int x^3\sqrt{1+x^4}\,\mathrm{d}x = \int \frac{1}{4}\sqrt{u}\,\mathrm{d}u = \frac{1}{6}u^{\frac{3}{2}} + C$$

$$\Rightarrow \int x^3\sqrt{1+x^4}\,\mathrm{d}x = \frac{1}{6}(1+x^4)^{\frac{3}{2}} + C$$

(d) Let $u = \ln x$. Then $\mathrm{d}u = \frac{1}{x}\,\mathrm{d}x$. Substituting, we get

$$\int \frac{1}{x(\ln x)^2}\,\mathrm{d}x = \int \frac{1}{u^2}\,\mathrm{d}u = -u^{-1} + C$$

$$\Rightarrow \int \frac{1}{x(\ln x)^2}\,\mathrm{d}x = -(\ln x)^{-1} + C$$

Examples

Solution of (e)

To find $\int \frac{x}{\sqrt{x+1}}\,\mathrm{d}x$, we let $u = x + 1$. Then $\mathrm{d}u = \mathrm{d}x$. Notice that there is an extra term "x" in the indefinite integral and we need to express it in terms of u i.e. $x = u - 1$. Therefore, we have

$$\int \frac{x}{\sqrt{x+1}}\,\mathrm{d}x = \int \frac{u-1}{\sqrt{u}}\,\mathrm{d}u = \int \frac{u}{\sqrt{u}}\,\mathrm{d}u - \int \frac{1}{\sqrt{u}}\,\mathrm{d}u$$

$$= \int u^{\frac{1}{2}}\,\mathrm{d}u - \int u^{-\frac{1}{2}}\,\mathrm{d}u = \frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + C$$

$$\Rightarrow \int \frac{x}{\sqrt{x+1}}\,\mathrm{d}x = \frac{2}{3}(x+1)^{\frac{3}{2}} - 2(x+1)^{\frac{1}{2}} + C$$

More examples

Example

Find each indefinite integral:

(a) $\int e^{2x}(1+e^{2x})^3\,\mathrm{d}x$

(b) $\int \frac{(\ln x)^3}{x}\,\mathrm{d}x$

(c) $\int \frac{x^2}{(x^3-2)^5}\,\mathrm{d}x$

(d) $\int \frac{e^x}{1+e^x}\,\mathrm{d}x$

(e) $\int \frac{e^{-1/x}}{x^2}\,\mathrm{d}x$